ARITHMETIC
IN
EPITOME:
OR, A
COMPENDIUM
OF ALL THE
RULES,
BOTH
VULGAR and DECIMAL.

WHEREIN
CLEAR and PLAIN DEMONSTRATIONS are deduced from the Principles of *Arithmetic* itſelf; without either Reference to *Euclid*, or Uſe of *Algebra*.

By Wm. WEBSTER, Writing-Maſter.

The TENTH EDITION,
Carefully Corrected, with ADDITIONS, by
ELLIS WEBSTER, *of the* Cuſtom-Houſe.

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To the RIGHT HONOURABLE

Sir *Robert Walpole*,

Knight of the Most Noble Order of the Garter, First Lord Commissioner of the Treasury, and Chancellor of the Exchequer, *&c.*

7-23-34 WCS

SIR,

THOUGH the Distrust so natural to a young Author, on his first Performance, did indeed discourage me from complying with my Inclination, in offering this attempt to your HONOR on its first Publication; yet, being a little encouraged by the favourable Reception its first Edition met with in the World, I did with all Humility presume to lay the second Impression at your Feet; well knowing the Advantage it would appear with under the Patronage of a NAME so eminent for the En-

couragement of *Clerkship*, and of every other Part of useful *Literature*.

Upon the third Revisal of this Work for the Press, I made such Additions and Improvements, as have, I hope, given it somewhat a better Title to so great a Protection; but still its Imperfections, I am very sensible, will be but too apparent to Your HONOR's Penetration: Yet that Goodness, of which I have already had so abundant Experience, gives me Hopes Your Honor will both excuse what may remain amiss, and forgive this renewed Presumption of,

Your HONOR'*s*

Most Obedient, and

Most Humble Servant,

William Webster.

PREFACE.

THE Indulgence with which the former Editions of this Treatise have been received, has encouraged me to endeavour the making it ſtill more worthy the Acceptance of the Public; not only by correcting the many Errors which had before eſcaped, but alſo by enlarging it with very conſiderable Additions: Having from the third Impreſſion added the *Theory* to the *Practic Part*, and endeavoured to inform the Reader's Judgment, as well as to aſſiſt his Memory.

And this I have attempted in a Manner very different from the ordinary Practice of Writers on this Subject, having never once quoted *Euclid*, nor perplexed the Learner with (to him unintelligible) *Algebraical* Demonſtrations.

ARITHMETIC and GEOMETRY are juſtly looked upon as the two main Pillars of the Mathematics, upon which the whole

Fabric is raiſed; but in comparing them, they are too often confounded: For, how great a Similitude ſoever there may be betwixt them, in ſome reſpects, yet, as their Subjects are different, ſo their Principles are certainly diſtinct, and each may be accounted for without References to the other. Such Compariſons may indeed be uſeful by way of Illuſtration, and give great Satisfaction to thoſe who have a competent Knowledge in both; but Concluſions from *Geometrical* Demonſtrations are beyond the reach of the young Arithmetician; and the Theory of *Lines* and *Diagrams*, as they are commonly made uſe of, can only ſerve to bewilder and confound him.

Algebra, indeed, is not properly a Science diſtinct from Arithmetic, but is only a different Method of Computation, performed by ſubſtituting *Letters* in the Place of *Figures*, and expreſſing the ſeveral Parts of the Operation by *Symbolical Characters*. 'Tis true, great Diſcoveries have been made in Numbers by this Means, and Arithmetic owes many of its Improvements to this admirable Invention: Beſides, it furniſhes us with a ſhort Way of arguing, and brings the Proofs of a Propoſition into a little Compaſs. But however uſeful it may be to thoſe who underſtand it, it is certainly an unin-

unintelligible Jargon to the mere Numerist, and can give him very little satisfaction when laid before him as a Demonstration.

Our knowledge in science should rise gradually from one step to another, and our references should be always backward to what went before; but as the useful parts of Arithmetic have been commonly taught by practical Rules, which are usually taken upon content, so little exactness is generally observed in the order of their delivery, as to their dependance upon each other: The reason of which is plain; for as all Affairs do not require the whole knowledge of Numbers, and as most common Business may be done without much skill in *Fractions*; so it is not only common, but reasonable enough, to carry the Learner thro' whatever may be performed by *Whole Numbers*, before he is entered upon the greater difficulties of *Fractional Parts*. However, the gradation mentioned should be observed as much as possible, especially in the Business of Demonstration; and the Maxims referred to for explanation should never be of a more abstruse nature than the thing to be explained.

I have therefore endeavoured to give the inquisitive Reader, who would understand the reason of things, all the satisfaction I

can; and have attempted to account for the Principles of Arithmetic from ſuch ſelf-evident Propoſitions, and natural conſiderations, as flow moſt immediately from Numbers themſelves, and fall within the Comprehenſion of ſuch who are the common Conſulters of *Arithmetical Tracts*; and who, it may be preſumed, are generally unacquainted with the more abſtruſe parts of the Mathematics.

My firſt Deſign in this Treatiſe was, according to its Title, to epitomize the Art: and to reduce to a *Pocket-ſize* what has ſometimes been ſwell'd to a *Folio.* Indeed what I firſt publiſhed was no more than a practical Memorandum, and, conſequently conſiſted only of Rules and Examples, with few or no Explications; but tho' the Inlargements are now very conſiderable, I have not yet gone beyond my Deſign; it is ſtill a Compendium, and far ſhort in bulk of other Treatiſes on the ſubject. I may therefore now recommend it to the ingenious *Clerks* and *Accomptants* of the ſeveral Offices of *Great Britain*, (for whoſe Uſe eſpecially it was at firſt deſign'd) and to ſuch other *Gentlemen* whoſe Buſineſs requires the practice of Arithmetic, with more confidence than before; it being very little increas'd in Size, tho' very much improv'd in Precept.

PREFACE.

As I ſuppoſe my Reader acquainted with the manner of performing the common Rules of *Addition*, *Subtraction*, *Multiplication*, and *Diviſion*; ſo I have made the work ſhort, by omitting ſuch needleſs Inſtructions: but the *Reaſons* of thoſe Operations (being the Fundamentals of Arithmetic) I thought neceſſary to lay down. The *Rule of Three* I have alſo endeavoured ſo to explain, that the Application of *Proportion*, as it runs thro' almoſt all other Rules, may be fully underſtood.

And for thoſe Practices of Arithmetic which will admit of no other than an *Algebraical* Demonſtration, being generally Parts more curious than uſeful, ſuch as are ſome propoſitions in *Progreſſion* and the *Double Rule of Falſe*; I have thought it ſufficient to give the practical Rules for their Performance, without their Demonſtration; ſince thoſe who are Proficients in *Algebra* cannot be thought to want ſuch Explanation; and thoſe who are not, cannot be ſuppoſed to underſtand it.

I have indeed ventured to account for the Extraction of the Roots, in a manner which ſeems ſomething contradictory to my profeſs'd Deſign: but when it is conſidered, that the Subject of thoſe Operations is entirely

geometrical, though the Operations themselves are *numerically* performed, it will be found that there is no other rational Method of Demonſtration, but that of *Lines*, which, however, I hope, I have ſo laid down, as to be clear and intelligible to every Reader of common ſenſe, who but underſtands the difference betwixt a *Line*, *a Surface*, and a *Solid*.

As to the Diſpoſition of the *Chapters*, they follow much in the ſame order as they are generally taught in *Schools*: For tho' I am ſenſible, that the Rules of *Practice* are not thoroughly to be underſtood without ſome knowledge of *Vulgar Fractions*, yet I have choſen to place Fractions in the laſt Chapter of *Vulgar Arithmetic*, becauſe (as I before obſerved) moſt common Buſineſs may be done by whole Numbers. So the Speculative Rules of *Progreſſion*, or *Arithmetical* and *Geometrical Proportion*, which, in a juſt *Theoretic* way of treating the ſubject of Numbers, ought to precede the *Rule of Three*, I have thought better to place juſt before *Vulgar Fractions*: however, *Progreſſion* and *Vulgar Fractions*, being each diſtinct Chapters, the Reader may look into them whenever he thinks it moſt proper.

The

PREFACE.

The ſecond Part of this Treatiſe, which contains the Doctrine of *Decimal Fractions*, &c. is a kind of Arithmetic peculiarly, as it were, adapted to the concerns of *Gentlemen*: I have therefore been more large than ordinary upon that Subject, and have run over the ſeveral Rules again, to ſhew its particular uſe and application. In the Chapter of *Intereſt* and *Rebate*, its excellency will moſt appear, every Calculation of that kind being performed to great advantage by Decimal Numbers. Under that head will likewiſe be found ſeveral uſeful *Tables* (which, that they might be perfectly correct, are exactly engraved on Copper) for the ready diſcovering the amount, and preſent worth both of ſingle Sums, and of Annuities. And as the Intereſt of Money is now at a lower Rate than when this Treatiſe was firſt publiſhed, I have in this Impreſſion added Tables, at 3 and 4 *per Cent.* to thoſe before publiſhed at 5 and 6. There are, beſides theſe, ſome other neceſſary *Tables*, interſperſed through this part of Arithmetic (which are alſo with the ſame exactneſs, printed from Copper-Plates); as thoſe for the reduction of the known parts of Coin, Time, Weight, and Meaſure, into decimal Parts of their proper *Integers*; and alſo that moſt uſeful *Ta-*

ble

ble, which ſhews the number of Days from any day in any Month to the ſame day in any other. The *Contents* will ſhew where to find any of theſe things; and in the Work itſelf are ſufficient Explanations both of their formation and uſe. I ſhall therefore add nothing farther by way of Preface, but ſubmit the whole Performance to the judgment and candour of the Public.

VULGAR ARITHMETIC,

IN

Whole Numbers and Fractions.

AS it is inconsistent with the intended brevity of this Treatise, to enter upon a nice enquiry after the first Inventors of Arithmetic, or by what degrees it has been raised to its present perfection; so neither is it my design, in the following pages, to initiate absolute Beginners; but supposing my reader acquainted, at least, with the methods of Adding, Subtracting, Multiplying, and Dividing, (tho' perhaps ignorant of the reasons thereof) I shall lay before him, in as few words as possible, the reasons of those Fundamentals, and their Application throughout all the varieties of practice.

Arithmetic (which is justly defined *The Art of Reckoning*) has for its subject, Number; and teaches us to give proper answers to all such questions as demand, *How many?* For the more ready performing of which, the ten following Characters, or Figures, have been invented and agreed upon by the greatest part of the known World, *viz.*

1 2 3 4 5 6 7 8 9 0

and by the various combinations and repetitions of these ten Characters, may every Number, how great soever, be easily and expeditiously expressed.

Thus much in general: I now proceed to particulars.

CHAP. I.

Of the five first and fundamental Rules, viz. *Numeration, Addition, Subtraction, Multiplication, and Division.*

Numeration, or Notation.

BY Numeration we learn the different value of Figures, by their different places; and, of consequence, to read or write any Sum, or Number.

The TABLE.

9	Units.	1
90	Tens.	12
900	Hundreds.	123
9000	Thousands.	1234
90000	X Thousands.	12345
900000	C Thousands.	123456
9000000	Millions.	1234567
90000000	X Millions.	12345678
900000000	C Millions.	123456789

From this Table may be observed:

1. The names of the several places, *viz* Units, Tens, Hundreds, &c. which proceed increasing by a tenfold proportion) from the right-hand to the left.

2. That every Figure hath two values; one in itself; the other from the place it stands in. Thus, on the left-side of the Table, the figure 9 in the upper line, standing in the unit's place, is only nine; but in the second

second line, being removed into the place of Tens, becomes ninety; and in the third line is nine hundred, &c.

3. That tho' a cypher is nothing in itself, yet it gives value to other figures, by removing them into higher places.

All which being very obvious, I proceed to the next Rule,

ADDITION.

BY Addition we find the whole, or total, of two or more parts, or sums.

In setting down the numbers to be added, care must be taken to place every figure in its proper column; that is, Units under Units, Tens under Tens, &c. Then will the reason of the work (the manner of which I suppose my reader well acquainted with) appear very evident from this undeniable Maxim, *viz.* *That the Whole is equal to all its Parts.* And the method of setting down the total may easily be accounted for from the nature of Numeration, which explains the different value of places, as they proceed from the right to the left hand: For as 9 is the greatest simple character, or figure; so every number exceeding 9, being compound, must require more places than one to express it. Thus the number 10 can no otherwise be expressed in figures, but by removing the figure 1 into the place of Tens, which is done by supplying the Unit's place with a cypher: And as it is the same with every other column (Ten being still the proportion of increase) consequently, when the sum of any column amounts to 10, or more, the units exceeding, if there be any, or a cypher, if none, must be set under such column; and the ten, or tens, of the amount, carried on as so many units, to the next column on the left.

What

What is here obſerved, as to carrying the Tens (the proportion of increaſe) from one column to another in Integers, may be as juſtly applied to the numbers we ſtop at, in adding ſums of different Denominations.

For your greater eaſe in caſting up of Money, learn the following Table.

PENCE-TABLE.

Pence.		s.	d.	Pence.		s.	d.
20	is	1	8	80	is	6	8
30		2	6	90		7	6
40		3	4	100		8	4
50		4	2	110		9	2
60		5	0	120		10	0
70		5	10				

Examples in whole Numbers, and Money.

℔.	*Yards.*	*l.*	*s.*	*d.*	
756	1325	735	18	09 ½	
132	4532	423	10	10 ½	
458	7345	784	12	05 ¾	
736	1298	297	08	04	
857	8473	542	11	11 ¼	
241	5249	298	14	07 ¾	
3180	28222	3082	17	00 ¾	*Total.*

Exam-

Examples in Avoirdupoise *and* Troy Weights.

AVOIRDUPOISE.

Tons.	*C.*	*q.*	lb.	*oz.*	*drams.*	
753	19	3	27	15	15	
347	08	1	17	10	06	
283	11	0	12	03	10	
549	05	2	13	09	07	
251	13	0	15	03	11	
2185	18	1	02	11	01	*Total.*

TROY-WEIGHT.

lb.	*oz.*	*dwts.*	*grs.*	
327	11	19	23	
429	10	17	19	
274	08	13	09	
478	10	12	05	
326	09	18	11	
1838	04	01	19	*Total.*
1510	04	01	20	*Tot. without the Top-line.*
1838	04	01	19	*Proof.*

To know how many Drams make an Ounce, or Grains a Penny-weight, &c. see the following Tables, Page 18.

Proof of Addition.

The proof of this Rule is usually by a second addition, without the top-line; which second sum, if, when added to the uppermost line, it makes the first total, the work is supposed right. *See the last Example.*

But

But it is as good, if not a better Proof, to produce the ſame total, by adding your columns both up and down.

SUBTRACTION.

Subtraction teaches us, by taking a leſſer number from a greater, to find the Remainder.

The reaſon of this Rule is evident from the ſame principles as Addition; Subtraction being but the reverſe thereof: And the number borrowed in any column (like what we ſtopt at in Addition) being always ſo many as would make 1 in the next.

Examples in Integers and Money.

	Yards.		l.	s.	d.
	7146325	*Lent*	812	13	08½
	1483972	*Paid*	190	19	10½
Rem.	5662353	*Rem.*	621	13	09¾
Proof.	7146325	*Proof*	812	13	08½

	l.	s.	d.
Borrowed at ſeveral Times.	429	11	08½
	212	17	10
	356	17	05¾
Borrowed in all	999	07	00¼
Paid	519	18	09¾
Rem.	479	08	02½
Proof.	999	07	00¼

AVOIR.

AVOIRDUPOISE.

Tons.	C.	q.	lb.	oz.	Drams.	
92	10	0	07	03	13	
14	13	3	11	12	05	
77	16	0	23	07	08	Rem.
92	10	0	07	03	13	Proof.

TROY-WEIGHT.

	lb.	oz.	dwts,	grs.	
From	672	10	05	09	
Take	139	11	05	21	
	532	10	19	12	Rem.
	672	10	05	09	Proof.

Proof of Subtraction.

Sums in this Rule are easily proved, by adding their remainders to their lesser numbers; which (if right) will make the greater.

MULTIPLICATION.

Multiplication (which is the fourth Rule) serves instead of many Additions; the product of a Multiplication being only the repetition of the Multiplicand so many times as there are units in the Multiplier.

Note. The ready performance of this and the next Rule, intirely depends upon the perfect knowledge of the following Table.

The

The TABLE.

3 times	3	9
	4	12
	5	15
	6	18
	7	21
	8	24
	9	27
4times	4	16
	5	20
	6	24
	7	28
	8	32
	9	36
5 times	5	25
	6	30
	7	35
	8	40
	9	45
6 times	6	36
	7	42
	8	48
	9	54
7 times	7	49
	8	56
	9	63
8 times	8	64
	9	72
9 times	9	81
12times	2	24
	3	36
	4	48
	5	60
	6	72
	7	84
	8	96
	9	108
	10	120
	11	132
	12	144

Three terms are uſed in Multiplication, *viz. Multiplicand*, *Multiplier* and *Product*.

The following Example ſhews to which line each term belongs.

EXAM-

EXAMPLE.

7563254138 *Multiplicand.*
23456789 *Multiplier.*

68069287242
60506033104
52942778966
45379524828
37816270690
30253016552
22689762414
15126508276

177409656468442882 *Product.*

1
8+8 *Proof.*
1

That Multiplication ſerves inſtead of many Additions, and conſequently, that the truth of its operation depends upon the ſame Reaſons, is eaſily proved: For ſuppoſe it were required to know the ſum of four ſevens; by Addition, the work will ſtand thus:

7
7
7
7

28

Too tedious a method for practice, being diſcoverable at once, by the Table of Multiplication to be 28.

Nothing therefore remains to be accounted for in this Rule, but the reaſon of placing every following particular product a place nearer to the left-hand than the product foregoing: But regard being ſtill had to the nature of Numeration, it will plainly appear, that as we begin with the right-hand figure of the Multiplier, or Unit's place; ſo the ſecond figure ſtanding in the place of Tens, the product thence ariſing, muſt, of con-

consequence, be ten times greater than the same figure, standing in the place of Units, would have produced; and so on, applying the same consideration to the produce of all the other figures in the Multiplier.

Proofs of Multiplication.

Multiplication is usually thus proved. Cast out the *nines* from the Multiplicand and Multiplier, and place the remainders on the right and left sides of a cross, thus made +. These two figures multiplied together, must have the *nines* cast out of their product, and the remainder placed at top: Then casting the *nines* also out of the product of your Multiplication, place its remainder at the bottom; which if it agrees with the figure at the top, the work is supposed right. *See the Example.*

A more certain Proof.

A Multiplication sum is then right, when the product, divided by the Multiplier, quotes the Multiplicand; or, divided by the Multiplicand, quotes the Multiplier.

ABBREVIATIONS.

(1*st*.) When either your Multiplicand, or Multiplier, or both, have one or more cyphers to the right-hand, only multiply by the significant figures, and set on the right-hand of the product so many cyphers as were in both the Multiplicand and Multiplier.

EXAMPLES.

527000	7583
4000	7000
2108000000	53081000

(2*dly*.) Therefore any number may be multiplied by 10, 100, 1000, &c. only by placing on the right-hand of

of it, one, two, three, or more cyphers; Thus 7295 multiplied by 10, is 72950; by 100, is 729500, &c.

(3dly.) To multiply any number by 5, add a cypher to it, and halve it. Or by 15, the same; and add both lines together.

EXAMPLES.

	5334 *multiplied*		
by 5 ——	53340	*by* 15 ——	53340
			26670
is ——	26670	*is* ——	80010

(4thly.) Any number may be multiplied by 11, 111, or 112, &c. as in the following Examples.

EXAMPLES.

7136 *multiplied by* 11
7136
———
78496

by 111 *thus*: 7136
7136
7136
———
792096

And by 112, *thus*: 7136
7136
7136
7136
———
799232

Or thus: 7136
112
———
85632
7136
———
799232

We now proceed to our fifth Rule.

DIVI-

DIVISION.

BY Division (which is the reverse of Multiplication) we find how often one number is contained in another.

In this rule there are also four terms, the *Divisor*, *Dividend*, *Quotient*, and *Remainder*.

EXAMPLE.

Divisor. *Dividend.* *Quotient.*

```
3456789)5678954367 83(164284
        22221653
         14809196
          9820407
          29058298
           14139863
             312707 Remainder.
```

As Multiplication serves instead of many Additions; so Division supplies the place of many Subtractions; as may be thus made evident. Suppose it were required to divide 28 by 7, that is, to find how often 7 is contained in 28; by Subtraction the work will stand thus:

```
28
 7
--
21
 7
--
14
 7
--
 7
 7
--
```

By

By which we find that 7 is 4 times contained in the number 28. But it may be difcovered by the rules of Divifion at one trial.

There are many methods of working this rule. The following Example is divided fix feveral ways.

Firft.

10(2
1008(1
42645(1332
32222
333

Second.

10(2
1008(1
32)42645(1332
32664
996

Third.

10(2
1008(1
32)42645(1332

Fourth.

21
8
10
10
32)42645(1332
32
96
96
64

Fifth.

32)42645(1332
32
106
96
104
96
85
64
21

Sixth.

32)42645(1332
106
104
85
21

Note.

Note, The two last, called the *Italian* ways, are most generally used.

Mr. *Alingham* has, indeed, set down nine ways ; but his three others are so very like the 1st, 2d, and 3d, of these, that they can scarce be called different.

ABBREVIATIONS.

(1*st*.) If there are any cyphers on the right-hand of your Divisor, you may cut off so many cyphers, or figures, on the right-hand of your Dividend; but remember to bring them down (if figures) to the remainder.

EXAMPLE.

```
21|00)8645|29(411
      84
      --
       24
       21
       --
        35
        21
        --
        1429
```

(2*dly*.) By the foregoing rule, you may observe, that to divide by 10, 100, 1000, *&c.* is only to cut so many figures from the right-hand of the Dividend, as there are cyphers in the Divisor.

EXAMPLE.

1|000)43682|735)

So the Quotient is 43682, *the Remainder* 735.

(3*dly*.) When your Divisor is 12, or consists only of one single figure, or can be reduced to one, by cutting off cyphers from its right hand, the work may be easily performed in one line, thus :

RULE.

RULE.

Drawing a line under the Dividend, set down under its first figure, how often the Divisor is contained in it; what remains imagine placed before the next figure; and, considering how often your Divisor is contained in the sum it makes, set down the number underneath, as before; and so proceeding through all the figures, set down what remains at last, in the Place where your quotient used to stand.

EXAMPLES.

4)93645(1	12)83675(11	7|00)5635|15(
23411	6972	805

If you are to divide several Numbers by one common Divisor (as in the calculating of Tables, &c.) that you may know exactly at once how often your Divisor will go, in some convenient Corner make a Table of your Divisor, by multiplying it severally by all the nine Digits: Thus, suppose 562 your Divisor:

562	1
1124	2
1686	3
2248	4
2810	5
3372	6
3934	7
4496	8
5058	9

Proofs of Division.

(1*st.*) Multiplication and Division mutually prove each other: For as if you divide the Product of a Multiplication by the Multiplier, the Quotient will be the Multiplicand; so, if you multiply the Quotient of

a Division by the Divisor, (taking in the remainder) the product will be the Dividend.

(2*dly*.) Another proof of Division is, by adding together those lines in the following example, marked with Asterisms (being the particular products of the Divisor, multiplied severally by each figure in the Quotient. together with the remainder of the Division) the total of which (if right) will be the Dividend.

(3*dly*.) Division may also be proved as Multiplication, by a cross, thus; casting out the nines from the Divisor and Quotient, place the remainders on its right and left sides; then multiplying the two figures so placed together, and casting the nines from the product, add what's left to the remainder of the Division; and still casting out the nines, let the overplus be placed at the top; then also casting the nines from the Dividend, set down the figure remaining at the bottom, which if it agrees with that at top, the work may be supposed right. See each proof in the following

EXAMPLE.

```
                736)863256(          1172
                    736*              736
                    ----             ----
    3               1272             7032
  7-+-2              736*           3516
    3               ----           8204
3d Proof.           5365           ------
                    5152*          862592
                    ----              664 Remainder.
                    2136           ------
                    1472*          863256 1st Proof.
                    -----
                     664*
                    ------
                863256 2d Proof.
```

Before we proceed to *Reduction*, it will be proper to insert the following Tables.

TABLES

TABLES *of* Engliſh *Coins, Weights, and Meaſures.*

Firſt, of COINS.

ACCOMPTS are kept in Pounds, Shillings, Pence, and Farthings, thus divided:

4 Farthings	make	1 Penny,	thus marked	*d.*
12 Pence	make	1 Shilling,	thus marked	*s.*
20 Shillings	make	1 Pound,	thus marked	*l.*

But the uſual Coins are,

			l.	*s.*	*d.*
Of Gold,	A Jacobus,	Value	1	5	0
	A Carolus,	Value	1	3	0
	A Guinea,	Value	1	1	0
	A ½ Guinea,	Value	0	10	6
Of Silver,	A Crown,	Value	0	5	0
	A ½ Crown,	Value	0	2	6
	The names of the reſt ſpeak their value, as a Shilling, a Six-pence, a Groat, or 4 *d.* a Three-pence, a Two-pence, a Penny.				
Of Copper,	A ½ Penny,	thus writ	0	0	0½
	A Farthing,	thus writ	0	0	0¼

Beſides the above-mentioned, we have ſtill in uſe the names of ſome other pieces, which are now but imaginary, *viz.*

		l.	*s.*	*d.*
A Mark,	Value	0	13	4
An Angel,	Value	0	10	0
A Noble,	Value	0	6	8

WEIGHTS.

TROY.

24 Grains	make	1 Pennywt.	thus mark'd	dwts.
20 Pennywts.		1 Ounce,		oz.
12 Ounces		1 Pound,		℔.

By *Troy Weight* are weighed Jewels, Gold, Silver, and all Liquors.

APOTHECARIES.

20 Grains	make	1 Scruple,	thus mark'd	℈
3 Scruples		1 Dram,		ʒ
8 Drams		1 Ounce,		℥
12 Ounces		1 Pound,		℔

By these Weights, Apothecaries compound their Medicines; but buy and sell their Drugs by *Avoirdupoise*.

AVOIRDUPOISE.

16 Drams	make	1 Ounce,	thus mark'd	oz.
16 Ounces		1 Pound,		℔
28 Pounds		1 quarter of a Hund.		qrs.
4 Quarters		1 Hundred,		C.
20 Hundred		1 Ton,		ton.

This is, at present, the common weight of *England*, by which Butter, Cheese, and all Groceries, &c. are weighed.

Note. One pound *Avoirdupoise*, is equal to 14 *oz.* 11 *dwts.* 15 *grs.* ½ *Troy*; and one ounce *Troy* is equal to 1 *oz.* 1 *dram*, and something above ½ *Avoirdupoise*.

WOOL-

WOOL-WEIGHT.

7 Pounds	make	1 Clove,	marked as written.
2 Cloves	make	1 Stone,	
2 Stones	make	1 Todd,	
6 Todds and ½	make	1 Wey,	
2 Weys	make	1 Sack,	
12 Sacks	make	1 Last,	

LEAD.

C.
19 ½ make a Fodder.

MEASURES.

WINE.

2 Pints	make	1 Quart,	thus mark'd *Qrt.*
4 Quarts	make	1 Gallon,	*Gall.*
63 Gallons	make	1 Hogshead,	*Hdd.*
2 Hogsheads	make	1 Pipe,	*Pipe.*
2 Pipes	make	1 Ton,	*Ton.*

BEER and ALE.

2 Pints	make	1 Quart,	thus mark'd *Qrt.*
4 Quarts	make	1 Gallon,	*Gall.*
9 Gallons	make	1 Firkin,	*Firk.*
2 Firkins	make	1 Kilderkin,	*Kild.*
2 Kilderkins	make	1 Barrel,	*Bar.*
3 Barrels	make	1 Butt,	*Butt.*

• *Note,* 8 Gallons make 1 Firkin of Ale.

DRY.

3 Pints	make	1 Gallon	thus mark'd	*Gall.*
2 Gallons		1 Peck,		*Peck.*
4 Pecks		1 Bushel,		*Bush.*
4 Bushels		1 Coom,		*Coom.*
2 Cooms		1 Quarter,		*Qr.*
5 Quarters		1 Wey,		*Wey.*
2 Weys		1 Last,		*Last.*

LONG.

3 Barley-Corns	make	1 Inch,	mark'd as writ.	* 1760 Yards make a Mile.
12 Inches		1 Foot,		
3 Feet		1 Yard *,		
5 Yards and ½		1 Pole or Perch,		
40 Poles		1 Furlong,		
8 Furlongs		1 Mile,		

LAND.

40 Square Perches	make	1 Rood.
4 Roods		1 Acre.

Note, That a Geometrical Pace is 5 Feet, and that there are 1056 such Paces in an *English Mile*.

CLOTH.

4 Nails	make	1 Quarter,	mark'd	*qr.*
4 Quarters		1 Yard,		*yd.*

Note, An Ell *Flemish* is 3 *qrs.* of a Yard: an Ell *English* is 5.

12	make	1 Dozen.
12 Dozen		1 Small Gross.
12 Small		1 Great Gross.

CHAP.

CHAP. II.

Of REDUCTION.

REDUCTION is but an application of Multiplication and Division : For,

First. All great names are brought into small, by multiplying with so many of the little ones as make one of the great.

Secondly, All small names are brought into great, by dividing by so many of the little ones as make one of the great.

Thirdly, To change one sort of money, or Weight, *&c.* into another, is only to bring both into one name, and to divide the one by the other.

EXAMPLES of each Sort.

(First Sort.)

	l. *s.* *d.*	
In	4295 12 3	how many Farthings ?
'tis multiplied by	20	*because* 20 s. *make* 1 l. Note, *The* 12 s. *are taken in*
	85912	
'tis multiplied by	12,	*because* 12 d. *make* 1 s. *And here the* 3 d. *are added.*
	1030947	
'tis multiplied by	4,	*because* 4 *Farthings make* 1 *Penny.*
Answer,	4123788	*Farthings.*

(*Second Sort.*)

In 4123788 Farthings, how many Pounds?

```
   4)4123788(
   ---------
  12)1030947(3
  --  -------
  2|0)8591|2(
      ----
       4295 l. 12 s. 3 d. facit.
```

We divide here by 4, 12, and 20, for the same reasons that we multiplied by them in the first example.

(*Third Sort.*)

Change 45967 *French* Crowns, at 4 s. 6 d. each into Guineas

```
                                      s.  d.
                                      4   6
                                      2
                                      --
              Six-pences in a   }
              French Crown.     }     9
21                 45967
 2                     9
--                 -----
42 { Six-pences in  42)413703(9850 Guineas.
   { a Guinea.         378
                        357
                        336
                        ---
                         210
                         210
                         ---
                           3 Six-pence.
```

More EXAMPLES.

In 85647 Guineas, how many Pounds Sterling?
Answer, 89929 *l.* 7 *s.*

In

In 59463 Marks, at 13 *s.* 4 *d.* how many Pounds?
Anſwer, 39642 *l.*

How many Dollars at 4 *s.* 4 *d.* are there in 89573 Piſtoles, at 17 *s.* 6 *d*?
Anſwer, 361737 Dollars, and 6 *d.* remain.

In 82 *lb.* 9 *oz.* 10 *dwts.* 5 *grs.* how many Grains?
Anſwer, 476885 Grains.

In 56 *Tons,* 13 *C.* 2 *qrs.* 21 *lb.* 7 *oz.* 11 *drams,* how many Drams?
Anſwer, 32505211 Drams.

How many minutes ſince the birth of our Saviour, it being 1766 years?
Anſwer, 925056000 Minutes.

How many Gallons are there in 7569 Tuns?
Anſwer, 1907388 Gallons.

Note, Theſe queſtions reverſed, will make as many more good examples.

CHAP.

CHAP. III.

Of Proportion, *or the* Rule *of* Three, *Direct, Indirect, and Double.*

THIS Rule is called the Rule of Three, because by three numbers given we find a fourth number ſought; which, when the Proportion is direct, muſt always bear the ſame ratio, or proportion, to the third number, as the ſecond bears to the firſt.

The chief difficulty of this Rule lies in ſtating its queſtions. For your direction threrefore obſerve, that, of the three given numbers, two always contain a Suppoſition, and the third a Demand.

The Number then on which the Demand lies, muſt always be the third in your ſtating; of the other two you will be ſure to find one of the ſame quality with the ſaid third; which being made your firſt, the number left, muſt, of conſequence, fall in the ſecond place, which ſecond number will alſo always be of the ſame kind with the fourth, or number ſought.

The queſtion being thus ſtated, you muſt (if they are not already ſo) bring your firſt and third numbers into one name, and your ſecond (if of ſeveral denominations) into its loweſt term; then multiplying your ſecond and third numbers together, and dividing the product by your firſt, the quotient will be the anſwer of your queſtion, in the ſame denomination you left your ſecond number. *See the Examples.*

EXAMPLES.

(Ex. 1ſt.) How many Ounces may I buy for 8 *l.* if I give after the rate of 2 *l.* for 18 Ounces?

	l.		*oz.*		*l.*
If 2 buy	—		18	—	what will 8?
			8		
			2)144(		

Answer, 72 *oz.*

(*Ex. 2d.*) What must I give for 122 Ells of Holland, if I pay after the rate of 34 *l.* for 138 Ells?

If 138 cost—34—what will 122?

```
                 34
                ---
                488
               366
              -----  l. s. d.
          138)4148(30  1  1¾
              414
              ---
                 8
                20
              ---
          138)160(1
              138
              ---
               22
               12
              ---
          138)264(1
              138
              ---
              126
                4
              ---
          138)504(3
              414
              ---
               90
```

The Remainder 8, *being Parts of a Pound, are multiplied by* 20, *to see what Shillings they will produce; and so the other Remainders by* 12, *and* 4, *to bring out the odd Pence and Farthings.*

This way of valuing the Remainder, is more fully explained in the seventh sort of Reduction of Vulgar Fractions.

(*Ex.*

(Ex. 3d.) How many Yards of Muslin can I buy for 42 *l.* 12 *s.* if $2\frac{1}{2}$ Yards come to 19 *s.* 6 *d?*

```
 s. d.        Yards              l.  s.
If 19 6 buy——2½——what will 42 12
    2          2                  20
   ——         ——                 ———
   39          5                 852
                                   2
                                 ———
                                1704
                                   5
                                 ———
                            39)8520(218
                               78
                               ——
       2)218(                  72
        ————                   39
 Answer, 109 Yards.            ——
                               330
                               312
                               ———
                                18
```

PROOF.

Of every four numbers in direct proportion, the product of the two means multiplied into each other, will be equal to the product of the two extremes so multiplied; therefore multiplying your first by your fourth number found, and comparing it with the produce of your second by your third, if the products agree, the work is right. Thus, in the first example, 72 (the fourth number found) multiplied by 2 (the first number) gives 144, equal to the produce of 8 times 18, the second and third numbers.

And from hence arises the invention of the foregoing rule, for finding the fourth number; for if the second number, multiplied by the third, be equal to

the

the first multiplied by the fourth, it is plain that if the product of the second and third be divided by the first, the Quotient must be the said fourth number; because every Dividend must be equal to the produce of its Divisor and Quotient, the Remainder (if any) being also considered

Sums in this rule may also be proved by a back-stating; thus reversing the third Example.

	l.	s.		yards.		s.	d.		yards.
If	42	12	will buy	109,	then	19	6	will buy	$2\frac{1}{2}$
	20			39		2			
	852			981		39			
	2			327					
	1704			1704)4251($2\frac{1}{2}$ yards.					
				3408					
				843					
				2					
				1686					
				18 } *Remainder of the other stating added.*					
				1704)1704(1					
				...0					

Though, in the former of the foregoing proofs, the manner of working the *Rule of Three* is sufficiently accounted for, and the reason of the operation plainly enough laid down; yet, as this may justly be looked upon as the main Rule of Arithmetic, and as the same application of proportion runs through almost all its other branches, it may, perhaps, help to make it still more evident, to look back again upon the first example, and observe it in another light. The question there stated was, If 2 *l.*—18 *oz.*—8 *l.* Now had it been, If 1 *l.*—18 *oz.*—8 *l.* 'tis clear, that if 1 *l.* would buy

buy 18 *oz.* then 8 *l.* would buy 8 times as many, that is, 144 *oz.* But then, as this is upon the supposition that 1 *l.* is the price of 18 *oz.* so if the price of the 18 *oz.* is doubled, that is, made 2 *l.* 'tis plain 144 *oz.* will then be twice the Quantity that 8 *l.* will buy, and must therefore be divided by 2, to give the true Answer. Again, had the supposed Price of 18 *oz.* been 4 *l.* then would 8 *l.* have bought but a fourth part of 144 *oz.* *viz.* 36 *oz.* consequently, in what proportion soever the first number increases, in the same proportion must the fourth number decrease; which is plainly effected by dividing by the first number.

Note, In the operation of the *Rule of Three*, the first and third numbers, after preparation, are no more regarded, as of any denomination; but are multiplied and divided with, only as absolute numbers, increasing and decreasing the middle number in just proportion.

Note also, That what has been said of the reason of working the *Rule of Three*, ought also to be considered in the operations of *Interest*, *Rebate*, *Fellowship*, *Exchange*, and all other applications of proportion.

We now proceed to some other practical observations.

1. If you would know at what rate you must sell out your goods by retail, so as to make a proposed gain by the whole, add the money you would gain to the sum the whole goods cost you; and then state your question thus: *If the whole be sold for the total of the Cost and Gain, what must any part?*

EXAMPLE.

Suppose I would, by the selling of 32 yards of Broad Cloth, which cost me 40 *l.* gain 5 *l.* for what must I sell it *per* yard? Then,

If

yards. *l.* *yard.*
If 32 be sold for 45, what will 1 ?

l.
40 *Cost.*
5 *Gain.*
—
45 *Total.*

1
— *l.* *s.* *d.*
32)45(1 8 $1\frac{1}{2}$ *facit.*
32
—
13
20
—
32)260(8
256
—
4
12
—
32)48(1
32
—
16
4
—
32)64(2
64
—
0

(*2d.*) Or if damage having happened to the Cloth, 5 *l.* were to have been lost by the whole, then the said 5 *l.* must have been subtracted from the cost, and the remainder made the second number, as before.

(*3d.*) If you would barter or exchange your goods for others, first find the value of your own, and then see what quantity of the others the sum will purchase.

EXAM-

EXAMPLES.

What quantity of Pepper, at 3 *s.* 6 *d.* per ℔. may I have in exchange for 426 C. of Tobacco, at 53 *s.* *per* C?

```
            C.       s.       C.
First, If   1 ------ 53 ------ 426
                                53
                              ----
                              1278
                             2130
                             -----
               Answer, 22578 Shillings.

          s. d.       ℔.        s.
Then, If  3  6 ------ 1 ------ 22578
          2                        2
          --                   -----
          7                  7)45156(6
                 Answer, 6450 Pounds 6/7.
```

ABBREVIATION.

The work of ſome ſtatings may be much ſhortened, by dividing their firſt and ſecond, or firſt and third numbers, by any figure, ſo that nothing may remain.

EXAM-

EXAMPLE.

l.	*Men.*	*l.*
If 192|00 *would pay* 5|00 how many would		17664 pay ?
32		2944
8		736
		5
		8)3680(

Answer, 460 *Men.*

The numbers of the above stating are abbreviated, by first cutting off the Cyphers from the first and second numbers, and then dividing the first and third numbers by 6 and 4.

The reasons of this abbreviation may be seen in the fifth sort of Reduction of Vulgar Fractions.

More Questions in the DIRECT RULE *of* THREE.

The cloathing of a Regiment of 740 men, comes to 3000 *l.* how much is that for each man ?

Answer, 4 *l.* 1 *s.* 0 *d.* $\frac{3}{4}$

How long shall I be laying up 10000 *l.* if I put by $\frac{1}{2}$ a Guinea a week ?

Answer, 366 *years*, 15 *weeks*, 4 *days*.

500 Seamen are to have 4 *d.* $\frac{1}{2}$ *per* day each, what will pay them for 23 months ?

Answer, 6037 *l.* 10 *s.*

What will an Estate of 4000 *l.* *per annum*, allow a Gentleman to spend a day ?

Answer, 10 *l.* 19 *s.* 2 *d.*

A Gentleman's daily expences are 13 *s.* 7 *d.* above which, he yearly lays up 600 Nobles, at 6 *s.* 8 *d.* each,

each, what is his Estate worth *per annum?*

Answer, 447 *l.* 17 *s.* 11 *d.*

A man owing 736 *l.* 10 *s.* compounds with his Creditors for 7 *s.* 9 *d. p. r l.* what will that amount to ?

Answer, 285 *l.* 7 *s* 10 *d.* $\frac{1}{2}$

If, at 5 *s. per* ell, I gain 8 *l per Cent.* by my Cloth, what shall I gain *per Cent.* if I sell the ell at 6 *s* 3 *d.*?

Answer, 35 *l.*

If Tobacco, which cost 13 *d. per* ℔. be sold for 18 *d. per* ℔. what is gained *per Cent ?*

Answer, 38 *l.* 9 *s.* 2 *d.* $\frac{1}{2}$.

Bought 5 pieces of Holland, each containing 56 ells *Flemish,* at 3 *s.* 2 *d. per* ell, what shall I gain in the whole, if I sell it for 5 *s,* 8 *d. per* ell *English ?*

Answer, 3 *l.* 5 *s.* 4 *d.*

I have by me 96 ℔. of Cloves, which cost me 58 *l.* but some damage having happened to them, I am willing to lose 8 *l.* in the whole; at what rate must I sell them *per* ounce ?

Answer, 7 *d.* $\frac{3}{4}$.

A Merchant sends over to *France* 482 tons of Lead, at 4 *l.* 10 *s. per* Fodder, *i. e.* 19$\frac{1}{2}$ *C.* what quantity of Wine, at 30 *l. per* Pipe, may he expect in return ?

Answer, 74 Pipes, 19 Gallons.

A Merchant sends to *Spain* 1300 pieces of Broad Cloth, each piece 47 yards, at 15 *s.* 6 *d. per* yard, to have returns from thence; the one half in Wine, at 65 *l. per* ton; and the other half in Oranges, at 3 *l.* 10 *s. per* Chest; what quantity of each will he have ?

Answer, { 364 Tons, 1 Hogshead of Wine.
6764 Chests of Oranges.

Two men part, at the same time, from the same place; the one travels north 32 miles a day, the other 36 miles a day south; how long will it be before they are 2000 miles asunder, supposing them to travel 12 hours each day ?

Answer, 29 days, 4 hours, 56$\frac{1}{2}$ minutes.

The

The Indirect Rule *of* Three.

IN the *Indirect Rule of Three*, the numbers are in reciprocal proportion, that is, the fourth number to be found, is to bear the same ratio to the second as the third does to the first, but in an inverted order; that is, the greater the third term is in respect to the first, the less must the fourth be in respect to the second.

This rule differs, in its operation, from the Direct, in that, after the question is stated, and the numbers of the statings prepared (as in the Direct Rule) your first and second must be multiplied together, and your third number be your Divisor. The Quotient, as before, will be the answer.

EXAMPLES.

(Ex. 1st.) What number of men must be employed, to finish, in 12 days, what 43 men would be 35 days about?

```
   days.   men.   days.
If 35 ——— 43 ——— 12
   43
   ——
   105
  140
  ————
12)1505
  ————
Answer, 125 Men.
```

(Ex. 2d.)

(*Ex. 2d.*) How many yards of Stuff 3 *qrs.* wide, will hang a Room which requires 420 yards of 5 *qrs.* wide?

	qrs.	*yards.*	*qrs.*
If	5 ——	420 ——	3
		5	
		3)2100(	
		Answer 700	

The reaſon of this operation will appear plain (after what has been ſaid in the Direct Rule) by conſidering the laſt example. Now it is clear, that if the Stuff, being 5 quarters wide, there are 420 yards required, then were the Stuff but 1 quarter wide, 5 times 420 yards, *viz.* 2100 yards muſt be allowed; conſequently, if the Stuff be 3 quarters wide, one third part of thoſe yards will be ſufficient; therefore 2100 divided by 3, will give the true anſwer required, *viz.* 700 yards.

To know whether a Queſtion belongs to the Direct *or* Indirect Rule *of* Three.

Obſerve, If the third number, being more than the firſt number, requires more, or, being leſs, requires leſs, it is *Direct*; but if the third number, being more, requires leſs, or being leſs, requires more, it is *Indirect.*

Or, without any regard to the diſtinction of *Direct* and *Indirect*; if *more* is required, let the leſſer of the two extremes be the Diviſor, if *leſs*, the greater.

More Questions in the Indirect Rule of Three.

If I lend *A* 136 *l.* for 3 months, how long must I keep 42 *l.* of his, to requite myself?

Answer, 9 months, 2 weeks, 6 days.

If 46 Clerks in 32 days finish a piece of writing, in what time would 55 Clerks accomplish the same?

Answer, 26 days, 9 hours, 9 minutes.

A Garrison, consisting of 1539 men, being besieged, have provisions only for 12 days; but it being necessary they should hold out 3 weeks, how many men must be sent out?

Answer, 660 men.

The DOUBLE RULE *of* THREE.

QUestions in this Rule have five numbers proposed, and are frequently answered by two statings, tho' they may be performed by one, as shall be shewn hereafter.

EXAM-

EXAMPLES.

(*Ex.* 1*st.*) The carriage of 32 hundred weight 56 miles comes to 12 *s.* After the same rate, what must I pay to have 78 hundred weight carried 94 miles?

```
            C.        s.       C.
   First, If 32 ------ 12 ------ 78
                                  12
                                 ----   s.  d.
                              32)936( 29  3 facit.
                                 64
                                 ----
                                 296
                                 288
                                 ----
       Miles    s. d.  Miles.    ..8
Then, If 56 ---- 29  3 ---- 94    12
                 12              ---
                ----           32)96(3
                 351              96
                  94              --
                ----               0
                1404
               3159
              ------
          56)32994(589        12)589(1
             280                -----
             ---              2|0|4|9(
             499              -------
             448        Answer, 2 l. 9 s. 1 d.
             ---
             514
             504
             ---
              10
```

Note,

Note, The solution had been the same, if the miles had been made first and third numbers of the first stating; and the *C.* weights the first and third numbers of the last.

Note also, this example may be done by one stating, thus:

C. s. C.
If 32——12——78
56 *Miles.* 94 *Miles.*

192 312
160 702

1792 7332
12
s. d.
1792)87984(49 . 1
7168 or
l. s. d.
16304 2 . 9 . 1
16128

.. 176
12

1792)2112(1
1792

. 320

(*Ex.* 2*d.*) How many men muſt be employ'd to reap 420 Acres in 17 days, if there were required 37 men to reap 54 acres in 5 days?

	Acres.	*Men.*	*Acres.*
Firſt, *If*	54 ——	37 ——	420
			37
			2940
			1260

54)15540(287
108
474
432
420
378
42

	Days.	*Men.*	*Days.*
Then, *If*	5 ——	287 ——	17
		5	

17)1435(84
136
75
68
7

Anſw. 84 *Men.*

Note, If you would work ſuch queſtions of the *Double Rule of Three*, as have one of their proportions indirect, by one ſtating, you muſt multiply the third number of your ſtating, by that number you would otherwiſe have placed under your firſt; and your firſt number by that you would have placed under the third; as in the following Example.

EXAM-

EXAMPLE.

```
Acres.    Men.    Acres.
If 54 ——— 37 ——— 420 )  The number of days
 * 17             5  }  which have relation to
 ———             ——— )  the 54 Acres.
  378           2100
  54              37
 ———           —————
  918          14700
                6300
                —————Men.
           918)77700(84 answer as before.
               7344
               —————
                4260
                3672
               —————
                 588
```

* *The number of days which have relation to the 420 Acres.*

Dr. *Harris*, in his *Lexicon Technicum*, teaches another way of ranging the five Numbers given in a *Double Rule of Three Question*; which, according to the following Rules, gives the solution, whether the proportion be direct or indirect. Take his method in his own words:

I. "Observing, that the given terms are always "five, whereof three are conditional and antecedent, "or supposititious, the other two demand the question, "and are consequents answering some of the former "antecedents; insomuch, that, with the answer, "there will be as many consequents as antecedents, "which must match one another in the same denomi-"nation exactly.

II. "For the right placing of the question and "terms, the three terms of the conditional part are "duly to be regarded: Let that which is the princi-"pal cause of Loss or Gain, Increase or Decrease, Ac-"tion or Passion, be put in the first Place; and that

 "which

" which betokeneth the ſpace of time, diſtance of " place, &c. be put in the ſecond place; and the re- " maining part in the third. The conditional part " thus ſtated, the other two terms, wherein the de- " mand lies, muſt be placed ſo under the former " terms, that they may correſpond one with another.

RULE 1.

" Then, if the blank, or place ſought, fall under the " third term, multiply the three laſt terms for a Divi- " dend, and the two firſt for a Diviſor; and the Quo- " tient gives the ſixth term required.

RULE 2.

" But if the blank fall under the firſt or ſecond " term, multiply the 1ſt, 2d, and 5th terms for a Di- " vidend, and the 3d and 4th for a Diviſor: The " Quotient gives the Anſwer.

EXAMPLE 1.

" If 12 Rods of ditching be done by 2 men in 6 " days, how many Rods ſhall be wrought by 8 men " in 24 days?

Anſwer, 192.

The numbers being ſtated as before directed, they will ſtand thus, the blank falling under the third place.

Men.	*Days.*	*Rods.*
2——	6——	12
8——	24	

Therefore, by the firſt Rule, the three laſt terms, *viz.* 12, 8, and 24, being multiply'd into each other, give 2304 for a Dividend; and the two firſt terms, *viz.* 2 and 6, multiply'd together, give 12 for a Diviſor, which quotes 192, the anſwer.

EXAM-

EXAMPLE 2.

If 2 men work 12 Rods in 6 days, how many men will work 192 Rods in 24 days?

Men.	*Days.*	*Rods.*
2 ——	6 ——	12
	24 ——	192

Here, according to the ſecond Rule, the firſt, ſecond and fifth terms multiply'd, give 2304, which divided by 288, the product of the third and fourth numbers, quotes 8 the anſwer.

More EXAMPLES.

If 400 pecks of Corn will ſerve 32 horſes 108 days, what quantity will 500 horſes eat in 20 days?

Anſwer, $1156\frac{2}{3}$ *pecks.*

If 56 gallons of Drink ſerve 25 perſons 120 days, how long will 200 gallons ſerve 12 perſons?

Anſwer, $892\frac{6}{7}$ *days.*

If a certain quantity of proviſion ſerves 30 men for 50 days with 24 ounces each, how much muſt 90 men have apiece when they are obliged to make the ſame proviſion ſerve them 60 days?

Ounces. *Drams.*
Anſwer, 6 —— $10\frac{2}{3}$.

Mr. *Gardiner* has publiſhed, in the explication of his Tables of Logarithms, a peculiar method of reſolving the ſeveral kinds of this Rule, (taken from the Papers of *W. Jones*, Eſq; F. R. S.) which is not only excellent in the uſe of Logarithms, but alſo in the way of common Arithmetic.

I. "Set down the terms expreſſing the condition of "the queſtion in one line, and in any order.

2. "Under each conditional term set its corre-
"sponding one, in another line.

3. "Multiply the producing terms of one line, and the produced term of the other line, continually, and the result for a dividend.

4. "Multiply the remaining terms continually, and let their product be the divisor.

5. "The quotient of this division will be the term required.

"By producing terms here is meant, whatsoever necessarily and jointly produce any effect: As the cause, and the time; length, breadth and depth; buyer and his money; seller and his goods; all necessarily inseparable in producing their several effects."

In a question where a term is understood, and not expressed, (as in the third Example) that term may be represented by Unity.

This Rule is so general, as to comprehend all cases that come under the common Rule of Three, whether direct, or inverse; whether single, or any how compounded; so that questions in any of the following Rules, where proportion is used, may be readily solved thereby.

EXAMPLES.

Ex. 1. Take the same as in page 38. Putting down the terms expressing the condition in the first line, and under each term its corresponding one in the second, filling the blank space (or term sought) with a capital Q thus.

	Men.		*Days.*		*Acres.*
If	37	require	5	to reap	54
	Q		17		420

Here

Here 'tis plain, that the produced terms are the acres, and the men and days are the terms producing the quantity of acres reaped; therefore looking in page 128 for the characters here used, and particularly the latter part of that for division, it will be that

$$Q = \frac{420+37+5}{54+17} = \frac{70+37+5}{9+17} = \frac{12950}{153} = 84 \text{ Men.}$$

Ex. 2. Take that in page 41, thus:

	Rods.	*Men.*	*Days.*
If	12 ——	2 ——	6
	Q	8	24

Here the produced terms are the rods, and the producing terms the men and days, as before.

$$Q = \frac{12+8+24}{2+6} = 8+24 = 192 \text{ Rods.}$$

Note, When any of the terms in the dividend are the same as in the divisor, they may be struck out in both; or when any in both may be equally divided, the quotients may be put instead of those terms, as in the first example.

Ex. 3. If 54 men can build a Fort in 18 days, when working 17 hours in each day; in how many days will 120 men build the same, in working but 12 hours each day?

	Men.	*Days.*	*Hours.*	*Fort.*
If	54 ——	18 ——	17 ——	1
	120	Q	12	1

Here the Fort being 1 is the term produced, and the men, days and hours, the producing terms; therefore

Ds. Hrs. Min.

$$Q = \frac{54+18+17}{120+12} = \frac{27+17}{40} = \frac{459}{40} = 11 : 5 : 42$$

Ex. 4. What is the Interest of 572 *l.* for 8 months, at 5 *l. per Cent. per Annum?* See page 68.

If 100 *l.*——5 *l.*——12 Months.
572 Q 8

Here the Intereſt is the term produced, and the principal and months the producing terms; therefore

$$Q = \frac{572+8+5}{100+12} = \frac{572}{30} = 19\tfrac{2}{30} = 19\,l.\ 1\,s.\ 4\,d.$$

Ex. 5. If 250 men in 5 days, working 16 hours each day, will dig a trench 240 yards long, 5 yards wide, and 4 deep; in how many days, working but 10 hours in each day, will 24 men dig a trench 360 yards long, 4 wide, and 3 deep?

Men.	*Days.*	*Hours.*	*long.*	*wide.*	*deep.*
If 250	5	16	240	5	4
24	Q	10	360	4	3

Here the produced terms are the trenches, conſiſting of length, width and depth, and the men, days and hours, the producing terms; therefore

$$Q = \frac{360+4+3+250+5+16}{240+5+4+24+10} = 75 \text{ days.}$$

As the Tables of Logarithms, when truly printed, are of excellent uſe to the proficients in Arithmetic; and having above an occaſion given for mentioning thoſe publiſhed by Mr. *Gardiner*, I think my readers ſhould be informed of their being the moſt correct, as well as complete, of any extant; and that he has publicly advertiſed ten ſhillings reward to be ſhewn one error, even of a logarithm not within the half unit of the loweſt place, or of a difference untrue between them, tho' given to every ten ſeconds of the Quadrant.

His Explication contains many curious ſolutions in logarithms, and very eaſy examples of getting the logarithm of any number, and the number to any logarithm, true to 19 places of figures, by tables therein given to 20 places; beſides thoſe of all numbers from 1 to 102100, with parts for two figures farther, every where in the ſame openings as the logarithms of the firſt five or ſix figures.

CHAP.

CHAP. IV.

Of PRACTICE, and TRETT and TARE.

BY PRACTICE Merchants compendiously cast up the Price of their Commodities.

And by TRETT and TARE, deduct their Allowances.

Both these Rules will be best explained by Examples; and both require the perfect Knowledge of the following

TABLE.

Of a Pound. The even Parts

s.	d.	
1	0	$\frac{1}{20}$
1	8	$\frac{1}{12}$
2	0	$\frac{1}{10}$
2	6	$\frac{1}{8}$
3	4	$\frac{1}{6}$
4	0	$\frac{1}{5}$
5	0	$\frac{1}{4}$
6	8	$\frac{1}{3}$
10	0	$\frac{1}{2}$

Of a Shilling. The even Parts

d.	
1	$\frac{1}{12}$
$1\frac{1}{2}$	$\frac{1}{8}$
2	$\frac{1}{6}$
3	$\frac{1}{4}$
4	$\frac{1}{3}$
6	$\frac{1}{2}$

Of a Pen.

Far.	
1	$\frac{1}{4}$
2	$\frac{1}{2}$

Of a Ton. The even Parts

C.	
2	$\frac{1}{10}$
$2\frac{1}{2}$	$\frac{1}{8}$
4	$\frac{1}{5}$
5	$\frac{1}{4}$
10	$\frac{1}{2}$

Of a Hund.

lb.	
14	$\frac{1}{8}$
16	$\frac{1}{7}$

Of $\frac{1}{2}$ C.

lb.	
7	$\frac{1}{16}$
8	$\frac{1}{14}$
14	$\frac{1}{8}$

Before we proceed to the rules of Practice, it will be proper to ſhew the manner of multiplying and dividing numbers of ſeveral denominations, by a ſingle figure, without Reduction.

Example of MULTIPLICATION.

	l.	*s.*	*d.*	
Multiply	5 .	8 .	4	*by* 8.
			8	
Product	43 .	6 .	8	

Here firſt, the 4*d.* multiplied by 8, gives 32 *d.* equal to 2 *s.* 8 *d.* therefore ſetting down 8 in the place of pence, we carry 2 to the produce of the ſhillings. Again, 8 times 8 *s.* being 64 *s.* and the 2 carried making it 66 *s.* or 3 *l.* 6 *s.* the 6 is ſet down in the place of ſhillings, and the 3 carried on to the Produce of the Pounds. Laſtly, 8 times 5 *l.* being 40 *l.* and the 3 carry'd making 43 *l.* the whole product appears to be 43 *l.* 6 *s.* 8 *d.*

Example of DIVISION.

	l.	*s.*	*d.*	
Divide	43 .	6 .	8	*by* 8.
8)	43 .	6 .	8 (	
Quotient	5 .	8 .	4	

In dividing the above ſum, we find, firſt, that 8 will go 5 times in 43 *l.* and that 3 will remain; therefore ſetting down 5 in the place of pounds, we multiply the ſaid remainder by 20, and taking in the 6 odd ſhillings, the produce is 66 *s.* in which again, the diviſor going 8 times, and 2 remaining, we ſet down 8 in the place of ſhillings, and multiply the 2 remaining by 12, which, with the 8 odd pence taken in,

in, produces 32 *d.* in which, lastly, the Divisor 8 going exactly 4 times, 4 is set in the place of pence, and the whole quotient will be found to be 5 *l.* 8 *s.* 4*d.*

Rules of PRACTICE, *with Examples.*

1. When the Price is such a number of pence as make an even part of a shilling, divide the quantity of goods by such part, and the quotient will be the answer in shillings.

EXAMPLE.

Suppose you would know the price of 5296 ounces of Gum, at 6 *d. per* ounce; then

$$\left.\begin{array}{l}6\,d.\ \textit{being}\ \tfrac{1}{2}\\ \textit{of a Shilling}\end{array}\right\}\ \begin{array}{l}5296\\ \hline 2648\ \textit{shillings is the Answer.}\end{array}$$

For were the price 12 *d.* or 1 *s.* the number of ounces would be the number of shillings they would cost: Therefore the price being 6 *d.* the ½ of a shilling, the answer must be but ½ so many shillings. So at 1 *d.* 1½ *d.* 2 *d.* 3 *d.* or 4 *d.* 'tis but the $\frac{1}{12}$, $\frac{1}{8}$, $\frac{1}{6}$, $\frac{1}{4}$, or $\frac{1}{3}$ parts of the quantity of the goods, and the quotient (as aforesaid) will be the answer in shillings; which divided by 20, gives the pounds.

Note, If any thing remains in dividing by the said parts (pence being the next denomination) it must be multiply'd by 12, the pence in 1 shilling; and the product divided again by the said part; and if any thing remains there, it must be multiply'd by 4, and still divided by the said part, to produce farthings.

EXAM-

EXAMPLE.

	oz.	d.
	5243	at $1\frac{1}{2}$ *per* Ounce.
d.		
$1\frac{1}{2}$ $\frac{1}{8}$	655	$4\frac{1}{2}$
Answer. - -	32*l.* 15*s.* $4\frac{1}{2}$	

2. If the pence of the price are not an even part of a shilling, it will require more than one division, to find the cost of the goods. Thus at 5 *d.* or any other number of pence between 6 and 12.

EXAMPLES.

d.	lb. 527 at 5 *per* lb.		d.	lb. 436 at 9 *per* lb.
$4\frac{1}{3}$	175	8	$6\frac{1}{2}$	218
$1\frac{1}{4}$	43	11	$3\frac{1}{2}$	109
	21|9	7		32|7
Answ.	10 *l.*	19*s.* 7*d.*	*Answ.*	16*l.* 7*s.*

In the first of these, 5*d.* is taken, thus: 4*d.* is the $\frac{1}{3}$ of a shilling, or the top-line, and 1*d.* the $\frac{1}{4}$ of 4 pence.

In the second, the $\frac{1}{2}$ of a shilling is first taken for 6*d.* and then the $\frac{1}{2}$ of that 6*d.* for 3*d.*

3. If the price is both pence and farthings; for the pence, as before; and for the farthings, take the even parts of a penny: thus,

EXAM-

EXAMPLE.

	oz.		d.
	532	at	$1\frac{1}{4}$ *per* Ounce.
1 *Penny*, $\frac{1}{12}$	44	.	4
1 *Farthing*, $\frac{1}{4}$	11	.	1
	5|5	.	5
Answer,	2*l*. 15*s*. 5*d*.		

4. When the price is more than 1 *s*. and less than 2 *s*. leave the top line for the shilling, and take your parts as before, for the remaining pence.

EXAMPLE.

d.		lb.		d.
		5437	at	$19\frac{1}{2}$ *per* lb.
6	$\frac{1}{2}$	2718	.	6
$1\frac{1}{2}$	$\frac{1}{4}$	679	.	$7\frac{1}{2}$
		883|5	.	$1\frac{1}{2}$
	Answer,	441 . 15*s*.		$1\frac{1}{2}$

5. If the price is two or more shillings, with pence, &c. either take the even parts of a pound from your top line for the shillings, which will give you the price in pounds; or multiply your top line by the number of shillings, which will give the answer in shillings, still working for the pence as before. See both ways in the example.

EXAM-

EXAMPLE.

		oz. s. d.	oz. s. d.
		5263 at 2 . 6 *per* Oz.	5263 at 2 . 6 *per* Oz.
s. d.			2
2 6	$\frac{1}{8}$	657 . 17 . 6	
d.			10526
6	$\frac{1}{2}$		2631 : 6
			1315|7 . 6

Answer, 657*l.* 17*s.* 6*d.*

By the first way, it is thus done: Had the price been 1*l.* or 20*s.* it is plain it would have come to as many pounds as there were ounces; therefore at 2*s.* 6 *d.* which is the $\frac{1}{8}$ of a pound, it must come to the $\frac{1}{8}$ of so much money.

The other is done by supposing the top line at 1*s.* then for 2*s.* it must be twice so much; and for the 6*d.* half the top line, or the value of a shilling, must be added. See both ways in another

EXAMPLE.

		lb. s. d.		lb. s. d.
		5276 at 19 9 *per* lb.		5276 at 19 9 *per* lb.
s.				19
10	$\frac{1}{2}$	2638		
5	$\frac{1}{2}$	1319		47484
4	$\frac{1}{5}$	1055 . 4	d.	5276
6d	$\frac{1}{8}$	131 . 18	$6\frac{1}{2}$	2638
3	$\frac{1}{2}$	65 . 19	$3\frac{1}{2}$	1319
Answer,		5210*l.* 1*s.*		10420|1

Answer, 5210*l.* . 1*s.*

6. When

6. When the price is any thing between one and two pounds, leave the top line for the pound, and take your parts for the remainder.

EXAMPLE.

s. *d.*	5384 at 22 . 6 *per* ℔.	
2 $6\frac{1}{8}$	673	
Answer,	6057*l.*	

7. When the price is two or more pounds, multiply the top line by the number of pounds; and for the remainder take parts, as before.

EXAMPLE.

		℔.	*l.* *s.* *d.*	
		2364 at 4 . 18 . $10\frac{1}{2}$ *per* ℔.		
		4		
s.		9456		
10	$\frac{1}{2}$	1182		
5	$\frac{1}{2}$	591		
2	$\frac{1}{5}$	236	8	
1	$\frac{1}{2}$	118	4	
6*d*	$\frac{1}{2}$	59	2	
3	$\frac{1}{2}$	29	11	
$1\frac{1}{2}$	$\frac{1}{2}$	14	15	6
Answer,		11687*l.*	00*s.*	6*d.*

8 When with the given quantity there are odd parts, such parts must be taken from the given price; as in the following examples.

EXAM-

EXAMPLES.

		C. qrs.	l. s.
		365 . 2	at 5 . 5 per C.
		5	
s.		1825	
5	$\frac{1}{4}$	91 . 5	
2 qrs.	$\frac{1}{2}$	2 . 12 . 6	
Answer,		1918*l.* 17*s.* 6*d.*	

		C. qrs. lb.	l. s. d.
		7365 . 3 . 14	at 8 19 6 per C.
		8	
s.		58920	
10 . .	$\frac{1}{2}$	3682 . 10	
5 . .	$\frac{1}{2}$	1841 . 5	
4 . .	$\frac{1}{5}$	1473 . 0	
6*d.* .	$\frac{1}{8}$	184 . 2 . 6	
2 qrs.	$\frac{1}{2}$	. . 4 . 9 . 9	
1 . .	$\frac{1}{2}$	. . 2 . 4 . 10$\frac{1}{2}$	
14 lb.	$\frac{1}{2}$	. . 1 . 2 . 5$\frac{1}{4}$	
Answ.		66108*l.* 14*s.* 6*d.*$\frac{3}{4}$	

As in the first of these, for the 2 *qrs.* $\frac{1}{2}$ the price of the *C.* is taken; so in the second, not only $\frac{1}{4}$ the price of the *C.* is taken for 2 *qrs.* but also $\frac{1}{2}$ the price of 2 *qrs.* for 1 *qr.* and $\frac{1}{2}$ that again for 14 lb. *See more Examples.*

EXAM-

EXAMPLES.

Tons. C. qrs. ℔. l. s. d.
573 . 13 . 2 . 18 at 9 . 12 . 10 per Ton.
9

		5157			
10s	$\frac{1}{2}$	286	10		
2	$\frac{1}{5}$	57	06		
10d.	$\frac{1}{12}$	23	17	6	
10 C.	$\frac{1}{2}$	4	16	5	
2	$\frac{1}{5}$		19	$3\frac{1}{4}$	
1	$\frac{1}{2}$		09	$7\frac{1}{2}$	
2 qrs.	$\frac{1}{2}$		04	$9\frac{3}{4}$	
14 ℔.	$\frac{1}{4}$		01	$2\frac{1}{4}$	
2	$\frac{1}{7}$			2	
2	$\frac{1}{7}$			2	

Answer, 5531*l.* 5*s.* 1*d.*$\frac{1}{4}$

℔. oz. dr. l. s. d.
527 . 13 . 15 at 2 . 8 . $7\frac{1}{2}$ per ℔.
2

		1054		
5s.	$\frac{1}{4}$	131	15	
2	$\frac{1}{10}$	52	14	
1	$\frac{1}{2}$	26	07	
6d.	$\frac{1}{2}$	13	03	06
$1\frac{1}{2}$	$\frac{1}{4}$	3	05	$10\frac{1}{2}$
8 oz.	$\frac{1}{2}$	1	04	$03\frac{3}{4}$
4	$\frac{1}{2}$		12	$01\frac{3}{4}$
1	$\frac{1}{4}$		03	$00\frac{1}{4}$
8 drs.	$\frac{1}{2}$		01	06
4	$\frac{1}{2}$			09
2	$\frac{1}{2}$			$04\frac{1}{2}$
1	$\frac{1}{2}$			$02\frac{1}{4}$

Answer, 1283*l.* 7*s.* 8*a.*

The

The last Sum another way.

		lb. oz. *drachms*	*l. s. d.*	
		527 13 15 at	2 8 $7\frac{1}{2}$ *per* lb.	
		48		
		4216		
		2108		
6 *d.*	$\frac{1}{2}$	263	6	
$1\frac{1}{2}$	$\frac{1}{4}$	65	$10\frac{1}{2}$	
8 *oz.*	$\frac{1}{2}$	24	$3\frac{3}{4}$	
4	$\frac{1}{2}$	12	$1\frac{7}{8}$	
1	$\frac{1}{4}$	3	$0\frac{1}{2}$	
8 *dr.*	$\frac{1}{2}$	1	6	
4	$\frac{1}{2}$		9	
2	$\frac{1}{2}$		$4\frac{1}{2}$	
1	$\frac{1}{2}$		$2\frac{1}{4}$	
		2566|7	8	

Answer, 1283*l.* 7*s.* 8*d.*

Another EXAMPLE.

		lb. *Troy. oz. dwts. grs.*	*l.*	*s.*	*d.*
		736 . 11 . 12 . 13 at	23	16	4 *per* lb.
		23			
		2208			
		1472			
10 *s.*	$\frac{1}{2}$	368			
5	$\frac{1}{2}$	184			
1	$\frac{1}{5}$		36	16	
4 *d.*	$\frac{1}{3}$		12	5	4
6 *oz.*	$\frac{1}{2}$		11	18	2
4	$\frac{1}{3}$		7	18	$9\frac{1}{3}$
1	$\frac{1}{4}$		1	19	$8\frac{1}{3}$
10 *dwts.*	$\frac{1}{2}$			19	10
2	$\frac{1}{5}$			3	$11\frac{3}{5}$
12 *grs.*	$\frac{1}{4}$				11[illegible]
1	$\frac{1}{12}$				[illegible]

Answer, 17552*l.* 2*s.* $9\frac{1}{2}$*d.*

Note,

Note. The foregoing examples, and all other questions in *Practice*, may be performed by the *Rule of Three*; *Practice* being only a compendious way of calculating such propositions in proportion, as have an unit for their first number: But to shew the brevity of this way of work, as well as to prove the truth thereof; see the last example done by the *Rule of Three*, as follows:

By the *Rule of Three* thus:

```
 lb.      l.  s.  d.            lb.   oz.  dwt  grs.
If 1 — cost 23 . 16 . 4 what will 736 . 11 . 12 . 13
 12       20                    12
 ——       ——                    ——
 12       476—                  8843
  20       12                     20
 ——       ——                   ——————
 240      5716                  176872—
  24                                24
 ——                            ——————
 960                            707491
 480                           353745
 ——                            ——————
 5760                          4244941
                                  5716
                               ————————
                               25469646
                               4244941
                              29714587
                              21224705        12
              5760)24264082756)4212 14(10
                              1274        2|0)351042
                              ..720  Answer, 17552. 2. 10¼.
                                1448
                                 2962
                                 ..827   Three Farthings
                                   2515   more than by
                                   2116   Practice lost there
                                      4   by the Fractions
                              ——————————  remaining.
                              5760)8464(1
                                 270
```

PROOF.

PROOF.

Sums in this rule may be proved either by taking the parts two ways, as the last example but one; or by the *Rule of Three*, as the last.

ABBREVIATIONS.

As this whole rule is indeed nothing else, it may be thought strange to mention short ways here, in a particular section; but as there are degrees of comparison, so even short ways may be more shortened.

1. The value of any quantity (the given price of one of which is an even number of shillings) may be most compendiously found, by multiplying the said quantity by ½ their number, doubling the Units of the first product for shillings, the rest being pounds.

EXAMPLE.

5276 *Ells of Holland*, at 8 *s. per Ell.*
4

Product, 2110 : 8

2. If the price is an odd number of shillings, it is but doing as before, and adding $\frac{1}{20}$ of the top line to the Product.

EXAMPLE.

Ells. *s.*
5278 at 15 *per Yard.*
7

3694 : 12
1 *s.* $\frac{1}{20}$ 263 : 18

3958*l.* 10*s.* *Answer.*

3. Were

3. Were there pence in the price, their parts might also easily be taken in the same method.

EXAMPLE.

	lb.	s.	d.
	5384 at	7 .	$7\frac{1}{2}$ *per* lb.
		3	
	1615 .	4	
1s. . $\frac{1}{20}$	269 .	4	
6d. . $\frac{1}{2}$	134 .	12	
$1\frac{1}{2}$. $\frac{1}{4}$	33 .	13	

Answer. 2052*l.* 13*s.*

4. Another short and useful way, when the quantity is small, is to multiply the price by it; thus, if your quantity is expressed by one figure, it is done by one line.

EXAMPLE.

	l.	*s.*	*d.*
5 *Yards of Velvet*, at	1 .	13 .	6 *per Yard.*
			5

Product, 8*l.* 7*s.* 6*d.*

5. So any quantity that can be expressed by 2 figures, and some of 3, may be easily and speedily done thus: Find out what two figures, multiplied together, will produce the given quantity, (as suppose 32 yards or pounds, &c. the given quantity which is produced by the multiplication of 8 by 4) and multiply the price by one of the said figures; and the product of that, multiplied again by the other, will give the answer.

EXAM-

EXAMPLES.

Yards.		*s.*	*d.*
32	at	15 .	8½ *per Yard.*
			8
The value of 8 yards,		6 . 5 .	8 —
which multiplied by			4

gives 25*l.* 2*s.* 8*d.* *the price of* 32 *yards,* —4 *times* 8 *being* 32.

Yards.		*l.*	*s.*	*d.*
144	at	1 .	5 .	6 *per Yard.*
				12
		15 .	6 .	0
				12

Product, 183*l.* 12*s.* 0*d.*

6. If no two figures will exactly produce those given, take such as will give the nearest product to them, which will generally be within one or two, more or less; the value of which must accordingly be added or subtracted.

EXAMPLE.

℔.		*l.*	*s.*	*d.*
55	at	1 .	10 .	9 *per* ℔.
				9
The price of 9 ℔.		13 .	16 .	9
which multiplied by				6
gives the price of 54 ℔		83 .	00 .	6
to which the price of 1 ℔. *added*		1 .	10 .	9
makes		84 .	11 .	3 *the value of* 55.

So,

So, had it been multiplied by 7 and 8, which would have given the value of 56 ℔. too much by 1 ℔. the price of 1 ℔. must have been subtracted.

7. If the quantity has odd parts, it will be easy taking the value of such parts from the given price: as in the following

EXAMPLE.

Yards.	*l.*	*s.*	*d.*
$43\frac{1}{2}$ at	2	12	7 *per Yard.*
			7
	18	8	1
			6
	110	8	6
qrs.	2	12	7
$2\frac{1}{2}$	1	6	$3\frac{1}{2}$
Product,	114*l.*	7*s.*	$4\frac{1}{2}$

TRETT *and* TARE.

TARE and TRETT are the allowances made to Merchants in buying their goods.

Tare, of what they can agree for *per* the whole, *per* chest, &c. or *per C.* for the weight of the bag, box, chest, &c. which contains the commodity.

Trett, for the waste, motes, or dust; and is always 4 ℔. *per* 104 ℔.

There is also another allowance sometimes given of 2 ℔. for every 3 *C.* for the turn of the scale, called cloff, or clough.

Note, the whole weight, before any allowances are made, is called *gross*; when part is deducted, the remainder is called *suttle*; but when all are taken from it, what is left, is called *neat*.

RULES.

RULES, with EXAMPLES.

1. The *Tare* of any quantity of goods may easily be found, if at so much in the whole, only by subtracting the said allowance from the gross weight; if at so much *per* chest, *&c.* by multiplying the pounds tare by the number of chests, *&c.* and subtracting as before; and if at so much *per C.* by taking such part or parts of the gross weight, as the allowance is of a *C.*

EXAMPLES.

First sort. Suppose 15 *C.* 2 *qrs.* 13 lb. tare were allowed on 456 *C.* 1 *qr.* 19 lb. of tobacco, what would be the neat weight?

	C.	qrs.	lb.	
From	456	1	19	*Gross.*
Subtract	15	2	13	*Tare.*
Remainder	440	3	6	

Second sort. What's the neat weight of 3 frails of raisins, each weighing 3 *C.* 2 *qrs.* 10 lb. gross; tare at 20 lb. *per* frail?

C.	qrs.	lb.		lb.
3	2	10		20
		3	*Frails.*	3 *Frails.*
10	3	2	*Gross.*	60 lb. or 2 *qrs.* 4 lb.
	2	4	*Tare.*	
Answer, 10	0	26	*Neat.*	

	C.	qrs.	lb.		lb.
Third sort.	246	3	12	*Gross.*	*Tare* 14 *per C.*
lb.					
$14\frac{1}{8}$	30	3	12	*Tare.*	
Answer,	216	0	0	*Neat.*	

Again,

		C.	*qrs.*	lb.	lb.
Again,		364 .	1 .	19 *Gross.*	*Tare* 16 *per C.*
	lb.				
	$16\frac{1}{7}$	52 .	0 .	$6\frac{1}{2}$ *Tare.*	
Answer,		312 .	1 .	$12\frac{1}{2}$ *Neat.*	

Note, 14 and 16 pounds may be called the standards of Tare; for from them may any other number of pounds, more or less, be taken, as in the following

EXAMPLES.

		C.	*qrs.*	lb.	lb.
		573 .	2 .	13 *Gross.*	*Tare* 17 *per C.*
	lb.				
	$14\frac{1}{8}$	71 .	2 .	$22\frac{1}{2}$	
	$2\frac{1}{7}$	10 .	0 .	27	
	$1\frac{1}{2}$	5 .	0 .	$13\frac{1}{2}$	
		87 .	0 .	7 *Tare.*	
Answer,		486 .	2 .	6 *Neat.*	

		C.	*qrs.*	lb.	lb.
		548 .	3 .	11 *Gross.*	*Tare* 10 *per C.*
	lb.				
	$16\frac{1}{7}$	78 .	1 .	$17\frac{1}{2}$	
	$8\frac{1}{2}$	39 .	0 .	$22\frac{3}{4}$	
	$2\frac{1}{4}$	9 .	3 .	$5\frac{1}{2}$ *Tare.*	
		49 .	0 .	$0\frac{1}{4}$	
Answer,		499 .	3 .	$10\frac{3}{4}$ *Neat.*	

Note, 'Tis first found what Tare 16 lb. would produce; because 8 lb. being the $\frac{1}{14}$ part of a C. would be a troublesome Divisor.

2. *Trett* being always 4 lb. *per* 104 lb. the constant method of finding it, is by taking the $\frac{1}{26}$ part of the line it is to be deducted from, 4 times 26 being 104. But it being difficult to divide by 26 in one line, it will be more safe and easy to do the work aside, as in the first Example.

EXAMPLES.

	C. qrs. lb.	lb.
	428 . 1 . 19 *Gross.*	*Trett* 4 *per* 104
lb. $4\frac{1}{26}$	16 . 1 . $25\frac{1}{2}$ *Trett.*	
facit	411 . 3 . $21\frac{1}{2}$ *Neat.*	

```
26)428(16
    168
     12
      4
26)49(1
     23
     28
    193
    193
     47
26)663(25
    143
     13
      4
26)52(2
      0
```

	C. qrs. lb.	lb.	
	836 . 2 . 17	Gross, *Tare* 22 *per* C.	
lb.		*Trett.* 4 lb. *per* 104 lb.	
16 $\frac{1}{7}$	119 . 2 . 2 $\frac{1}{4}$		
4 $\frac{1}{4}$	29 . 3 . 14 $\frac{1}{2}$		
2 $\frac{1}{2}$	14 . 3 . 21 $\frac{1}{4}$		
	164 . 1 . 10	*Tare.*	
	672 . 1 . 7	*Suttle.*	
lb.			
4 $\frac{1}{26}$	25 . 3 . 12	*Trett.*	
facit	646 . 1 . 23	*Neat.*	

3. *Clough* (which is always 2 lb. for 3 *C.*) may be found by taking the $\frac{1}{168}$ part of the line it is to be deducted from, 2 lb. being the $\frac{1}{168}$ part of 336 lb. or 3 *C.* But let the work be done aside, as directed in finding the Trett. Or the allowance for *Clough* may be found, by first dividing the hundreds of the line it is to be taken from, by 3, which brings them into 3 C's; then 2 lb. being to be allowed for every 3 *C.* as many 3 *C*'s as it produces, so many 2 lb's it will allow; then dividing by 56 (the double pounds in a hundred) the quotient will be hundreds, and the remainder double pounds; to which adding, what may be allowed for the odd hundreds, quarters, and pounds of the given weight, it makes the whole *Clough*; which subtracted from the said given weight, leaves the Neat. See both ways in the first Example.

EXAMPLE 1.

What will be the neat Weight of 5647 *C.* 3 *qrs.* 13 lb. *Gross*, allowing for Clough 2 lb. for 3 *C*?

```
  C.    qrs.  lb.
5647  .  3  . 13 Gross.
  33  .  2  . 13 Clough.
-------------------
5614  .  1  . 00 Neat.
```

```
          C.
168)5647)33              3)5647(1
    ----                   ----
    .607             56)1882(33 . 2 . 12
    ----                ----
     103                 202
       4                 ----
    ----- qrs.        14)34(2
168)415(2                --
    ----                 .6
    .79
     28
    ----
     635
    159
    ---- lb.
168)2225(13
    ----
    .545
    ----
     .41
```

Note, The remainder 34 (which are double pounds) is divided by 14, the double pounds in a *qr.* to bring the said remainder into quarters of an hundred, making 2 *qrs.* and 12 lb. And 1 lb. being allowed for the odd *C.* 3 *qrs.* and 13 lb. the whole makes, as before, 33 *C.* 2 *qrs.* 13 lb.

EXAM-

EXAMPLE 2.

What's the neat weight of 3 hogsheads of tobacco, weighing, *viz.*

	Number.	C.	*qrs.*	lb.		
	1 ——	5	3	18		
	2 ——	4	2	11		
	3 ——	5	1	19		
		15	3	20	*Gross.*	*Tare* 7 lb. *per* C. *Trett* 4 lb. *per* 104 lb. *Cough* 2 lb. for 3 C.
lb.						
$14\frac{1}{8}$		1	3	27		
$7\frac{1}{2}$			3	$27\frac{1}{2}$	*Tare.*	
		14	3	$20\frac{1}{2}$	*Suttle.*	
$4\frac{1}{26}$			2	$8\frac{1}{4}$	*Trett.*	
		14	1	$12\frac{1}{4}$	*Suttle.*	
				$9\frac{1}{2}$	*Clough.*	
	Answer,	14	1	$2\frac{3}{4}$	*Neat.*	

C.
3)14(4
——
2

Note, The 4 three hundreds allow ——— 8 lb.
And the odd 2 C. 1 *qr.* 12 lb. $\frac{1}{2}$ allow ——— $1\frac{1}{2}$

Total Clough $9\frac{1}{2}$

CHAP. V.

Of INTEREST.

INTEREST is either	Simple,	which arifes both from the Principal.
	Or	
	Compound,	which arifes only from Principal and Intereft.

SIMPLE INTEREST.

SIMPLE INTEREST being one of the branches of Proportion, or the *Rule of Three*, is worked as follows:

1. The Intereft of ony Sum for one year is found by this plain proportion, *viz.*

As 100*l.*
To the rate of Intereft:
So is the given Principal,
To the Intereft required.

EXAMPLE.

What's the Intereft of 429 *l.* for a year, at 6*l. per Cent. per Annum?*

	l.		*l.*		*l.*
If	100	———	6	———	429

```
          429
            6
         ————
        25|74
           20
         ————
        14|80
           12
         ————
         9|60
            4
         ————
         2|40
```

Answer, 25 *l.* 14 *s.* 9*d.*½

Note, The cutting off two figures to the right-hand, divides such lines by 100. *See Abbreviations of Division.*

2. To find the Interest of any sum for 2 or more years, multiply the year's amount by the number of years required.

EXAMPLE.

What interest will the foregoing principal produce in 4 years, at the same rate of interest?

	25	14	9½	*the interest for one year.*
		4		
Product	102	19	2	

3. To find interest for months, divide the year's amount by the even parts they make of a year.

EXAMPLE.

What's the intereſt of 572 *l.* for 8 months, at 5*l.* *per Cent. per annum?*

l. *l.* *l.*		*l.* *s.*
If 100——5——572		28 . 12
5	M.	
28|60	$6\frac{1}{2}$	14 . 6
20	$2\frac{1}{3}$	4 . 15 . 4
12|00		*Anſwer,* 19*l.* 1*s.* 4*d.*

4. If the time is weeks or days, inſtead of taking the even parts of a year as for months, it muſt be done by a ſecond ſtating in the *Rule of Three.*

EXAMPLES.

Ex. What's the intereſt of 321*l.* 16*s.* 8*d.* for 3 weeks, at 6*l.* *per Cent. per annum?*

l. *l.* *l.* *s.* *d.*
If 100——6——321 . 16 . 8
6

19\|31 . 00 . 0
20

6\|20
12

2\|40
4

1\|60

Anſwer 19*l.* 6*s.* 2*d.* $\frac{1}{4}$ *for a year.*

Then,

	Weeks.	l.	s.	d.	Weeks.
Then, *If*	52 ——	19 .	6 .	$2\frac{1}{4}$ ——	3

```
                        3
        ——————————————   l.  s.  d.
  52)57 . 18 . 6¾(1 . 2 . 3¼
     —
     5
    20
    ——
 52)118(2
    ——
    14
    12
    ——
 52)174(3
   18
    4
   ——
 52)75(1
   ——
   23
```

Ex. 2. What interest is now due on a bond of 420*l.* 10*s.* commencing interest at 5*l.* 10 *s. per Cent. per annum, July* the 10th, 1766; this being *March* the 25th, 1767?

```
  l.       l.   s.       l.      s.
If 100 —— 5 . 10 —— 420 . 10
                              5
                   s.  ————————————
                       2102 . 10
                 10½    210 .  5
                       ————————————
                       23|12 . 15
                           20
                       ————————————
23 l. 2 s. 6 d. ½       2|55
for a Year.                12
                       ————————————
                        6|60
                           4
                       ————————————
                        2|40
```

July	21
August	31
September	30
October	31
November	30
December	31
January	31
February	18
March	25
	258

 Then,

Days. l. s. d. Days.
Then, *If* 365—23 . 2 . 6½——258
20
———
462
12
———
5550
4
———
22202
258
———
177616
111010
44404
——— 4
365)5728116(15693(1
365 ——— 12)3923(11
——— ———
2078
1825 2|0)32(6
——— ———
2531 16*l.* 6*s.* 11*d.*¾ *for* 258 *days.*
2190 23*l.* 2*s.* 6*d.* ½ *for a year.*
——— ———
3411 39*l.* 9*s.* 5*d.*¼ *total Interest.*
3285
———
1266
1095
———
171

Another way to cast up Interest, *when the principal has odd money in it, and the time, weeks, or days.*

RULE.

Bring your principal into its lowest denomination, and multiply it by the number of days; and if at 5 *l.*

per Cent. divide the product by 7300, the quotient gives the answer in the same denomination the principal was reduced into. *Note,* The number 7300 is found by multiplying 365 by 100, and dividing by 5.

EXAMPLES.

What's the interest of 432 *l.* 12 *s.* 6 *d.* for 8 days, at 5 *l.* *per Cent. per annum?*

```
          l.    s.   d.
          432 . 12 . 6
             20
          ----
          8652
            12
        ------
        103830
             8
        ------      12
73|00)8306|40(113
      73            ----
     ----         9 s. 5 d. 3/4 Answer.
      100
       73
      ----
      276
      219
      ----
      5740
         4
73|00)229|60(3
      219
      ----
       10
```

If you would cast up interest at 6 *l* *per Cent.* by this rule, you must add ⅕ of the quotient to the interest produced at 5 *l.* *per Cent.* and so at other rates, it

it is but adding or subtracting such and such parts of the quotient to or from what it produces at 5 *l.* For instance, suppose the last question done the common way, for an

EXAMPLE.

```
             l.    s.
            420 . 10 Principal.
             20
           ----
           8410
            623 number of days.
          -----
          25230
         16820
        50460
       -------          s.     d.
73|00)5239|430(717  .  8¼
       511         s.
      ----    10 1/10  71 . 9¼
       129   -----------------
        73       78|9 . 6
      ----
       564   l. 39|9 . 6   Answer, 1 farthing more than
       511                  by the other way.
      -----
       5330
         12
      -----
73|00)639|60(8
       584
      -----
        5560
           4
      ------
73|00)222|40(3
       219
      -----
         340
```

Note,

Note, The Rate here being 5 *l.* 10 *s.* and 10 *s.* being $\frac{1}{10}$ of 5 *l.* there is therefore added $\frac{1}{10}$ of the quotient, or the amount, at 5 *l. per Cent.*

More Examples in Simple Interest.

At 6*l. per Cent.* what will 589 *l.* come to in a year?

Answer, 35 *l.* 6 *s.* 9 *d.* $\frac{1}{2}$.

What will the same sum, at the same rate, amount to in 8 years?

Answer, 282 *l.* 14 *s.* 4 *d.*

What's the interest of 7020 *l.* for 5 months, at 5 *l. per Cent. per annum*?

Answer, 146 *l.* 5 *s.*

What's a week's interest of 1020 *l.* at 7 *l. per Cent*?

Answer, 1 *l.* 6 *s.* 11 *d.*

What will the interest of 5429 *l.* 10 *s.* 8 *d.* come to in 20 days, at 5 *l. per Cent*?

Answer, 14 *l.* 17 *s.* 6 *d.*

What interest is now due on a Navy-bill, commencing interest at 6 *l. per Cent. per annum*, *June* the 4th, 1765, this being *January* the 10th, 1767, the principal being 836 *l*?

Answer, 80 *l.* 7 *s.* 10 *d.*

Compound Interest.

Compound Interest, as was before said, arises both from the principal and interest; (*i.e.*) when interest on money becoming due, and not paid, the same rate is allowed on the unpaid interest, as was before on the principal.

There-

Therefore, to work sums in this rule, after having found the first year's interest, as before, add it to the principal, and find the interest of the sum; and so continue to add every year's produce, still accounting the sum a new principal.

EXAMPLE.

What will be the amount of 400 *l.* forborn 3 years and ½, at 6*l. per Cent. per annum*, compound Interest?

	l.		*l.*		*l.*	
If	100	——	6	——	400*l.*	*Principal 1st year.*
					6	

l. 24|00

	l.		*l.*		*l.*	
If	100	——	6	——	424	*Principal 2d year.*
					6	

l. 25|44
20

s. 8|80
12

d. 9|60
4

2|40

If

l.
424
25 . 8 . 9½

If 100——6——449 . 8 . 9½ *Princip. 3d year.*
6

l. 26|96 . 12 . 9
20

s. 19|32
12

d. 3|93
4

3|72

l. s. d.
449 . 8 . 9½
26 . 19 . 3¾

If 100 —6——476 . 8 . 1¼ *Princip. 4th year.*
6

l. s. d. l. 28|58 . 8 . 7½
28 11 8 20

M.
6½ 14 5 10 11|68
476 8 1¼ 12

490 13 11¼d. 8|23
Total amount. 4

94

Or

Or, more plain :

Interest the 1st Year	24 .	0 .	0
2d ——	25 .	8 .	9½
3d ——	26 .	19 .	3¾
½ *Year*	14 .	5 .	10
Total Interest	90 .	13 .	11¼
Principal	400 .	0 .	0
Total amount	490 .	13 .	11¼

More EXAMPLES.

What will 1000 *l.* amount to in 3 years, at 6 *l. per Cent. per annum*, Compound Interest?

Answer 1191*l.* 0*s.* 3*d.* ¾

At 5*l.* 10*s. per Cent. per annum*, Compound Interest, what will 560*l.* 10*s.* 9*d.* amount to, forborn 5 Years and a half?

Answer, 752*l.* 14*s.* 10*d.* ½.

CHAP.

CHAP. VI.

Of REBATE, *or* DISCOUNT.

REBATE, or DISCOUNT, is the abating so much money on a debt paid before it is due, as might be gained again by the money received, if put out to interest at the same rate, and for the same time. So 100*l.* present money would discharge a debt of 106 *l.* due at a year to come; rebate being made at 6*l. per Cent.* because 100*l.* put out to interest for a year, at the said rate, would regain the 6 *l.*

To work Sums in this RULE.

First, find the interest of 100 *l.* for the time; then by a *Rule of Three* stating, of which the first number must be 100 *l.* with the interest found; the second number, the interest alone (or 100 *l.* alone, if you would find the present Money); and the third, the debt, or Sum propounded, you will find the answer.

EXAM-

EXAMPLES.

Ex. 1. What's the rebate of 420 *l.* for a year, at 6 *l. per Cent. per annum?*

```
If 106——6——420
                6
               ——— l.
         106)2520(23
             212
             ———
              400
              318
              ———
               82
               ——
               20
              ——— s.
Answer, 23 l. 15s. 5d.½   106)1640(15
              106
              ———
              580
              530
              ———
               50
               12
              ——— d.
          106)600(5
              530
              ———
               70
                4
              ———
          106)280(2
              212
              ———
               68
```

Ex. 2.

Ex. 2. What present money will discharge a debt of 732 *l.* due at 3 months, rebate being made at 6 *l. per Cent. per annum*?

l.		*l.*
6 *Interest for a year of*		100
Months—		
$3\frac{1}{4}$	1 . 10	

l. *s.* *l.*
Then, *If* 101 . 10 *fall to* 100, *what will* 732 ?

```
      2                                 2
    ----                             ------
     203                          203)146400(721
                                       1421
                                       ----
                                       ..430
                                        406
                                       ----
                                        .240
                                         203
                                       ----
                                         .37
                                          20
                                      -------
                                      203)740(3
                                          609
                                          ---
                                          131
                                           12
                                       -------
                                      203)1572(7
                                          1421
                                          ----
                                          .151
                                             4
                                        ------
                                      203)604(2
                                          406
                                          ---
                                          198
```

Answer, 721 *l.* 3 *s.* 7*d.* $\frac{1}{2}$

Ex.

Ex. 3. What rebate must be allowed on a Bill of Exchange for 85*l.* 10*s.* due *September* the 8th, this being *July* the 4th, rebate being made at 5 *l.* *per Cent.* *per annum*?

			Days.	*l.*	*Days.*
		If	365 ——	5 ——	66
July	27				5
Aug.	31				
Sept.	8				330
					20
	66				

$$
\begin{array}{r}
\;\; s. \quad d.\\
365)6600(18 \;.\; 0\tfrac{3}{4}\\
365\\
\hline
2950\\
\hline
..30\\
12\\
\hline
360\\
4\\
\hline
365)1440(3\\
1095\\
\hline
345
\end{array}
$$

Then,

```
              l.     s.    d.     s.    d.    l.    s.
Then, If 100 . 18 . 0¾ — 18 . 0¾ — 85 . 10
          20                12           20
       -----              -----        -----
        2018                216         1710
          12                  4           12
      ------              -----       ------
       24216                867        20520
           4                               4
      ------                          ------
       96867                           82080
                                         867
                                     -------
                                      574560
                                     492480
                                    656640
                                   --------     4
                          96867)71163360(734(2
                                678069      ----
                                ------- 12)183(3
                                 335646     ----
                                 290601    15s. 3½
                                 ------
                                 450450
Answer, 15s. 3d. ½               387468
                                 ------
                                  62982
```

Ex. 4. A Bill of Exchage for 250 *l.* is dated at *Amsterdam*, *June* the 14th, N. S. at usance is accepted, and payment offered the 15th *ditto*, O. S. what must be then received, rebate being made at 6 *l.* *per Cent.* *per annum*?

Note, New Style is 11 days before Old Style.

Usance signifies the usual time of payment of bills between one country and another; which between *England* and *Holland* is one month; double Usance, two months, &c. *Note also*, Merchants of *London* generally

rally allow 3 days beyond the time appointed, calling them 3 days of grace. Therefore the bill being dated *June* the 14th N. S. or the 3d O S. and accepted at usance, or a month, it becomes due *July* the 3d, O S. but 3 days of grace being allowed, makes it the 6th. So if the money is paid the 15th of *June* O. S. rebate must be made for 21 days. Therefore,

```
     Days.        l.        Days.
  If 365 ———————— 6 ———————— 21
                              6
                            ————
                            126
                             20
                            ————  s.
                        365)2520(6
                            2190
                            ————
                            .330
                              12
                            ————  d.
facit 6s. 10d. 3/4      365)3960(10
                             365
                            ————
                            .310
                               4
                            ————
                        365)1240(3
                            1095
                            ————
                            .145
```

If

```
     l.   s.  d.       l.        l.
If 100 . 6 . 10¼ —— 100 ——— 250
        20                       20
      ————                     ————
      2006                     5000
        12                       12
    ——————                   ——————
     24082                    60000
         4                        4
    ——————                   ——————   l.
     96331           96331)24000000(249
                           192662
                           ——————
                           .473380
                            385324
                           ———————
                            .880560
                             866979
                            ———————
                             .13581
                                 20
                            ———————  s.
                      96331)271620(2
                            192662
                            ——————
                            .78958
                                12
                            ——————  d.
                      96331)947496(9
                            866979
                            ——————
Answer, 249 l. 2s. 9d. ¾    .80517
                                 4
                          ——————— far.
                      96331)322068 (3
                            288993
                            ——————
                            .33075
```

More

More EXAMPLES,

What's the rebate of 100 for ½ a year, at 6 *l. per Cent. per annum*?

Answer, 2 *l.* 18*s.* 3*d.*

What's the rebate of 538 *l.* 10 *s.* for 5 months, at 5 *l. per Cent. per annum*?

Answer, 10 *l.* 19 *s.* 9 *d.* ½

What present money is a debt of 629 *l.* due at 3 weeks, worth, rebate being made after the rate of 6*l.* 10 *s. per Cent. per annum*?

Answer, 626 *l.* 16 *s.* 7 *d.* ½

What present money will discharge 10000 *l.* due at 5 days, rebate being made at 8 *l. per Cent. per annum*?

Answer, 9989 *l.* 1 *s.* 5 *d.* ¼

CHAP.

CHAP. VII.

Of EQUATION *of* PAYMENTS.

THE design of this rule is, when several sums are due at several times, to find a mean time for paying the whole debt.

The common way to do which is,

Multiply each sum by its respective time, and add all the products together, dividing their total by the whole debt; the quotient is accounted the mean time of paying the whole debt.

EXAMPLE.

A owes to *B* 40 *l.* to pay at 3 months, 60 *l.* at 5 months, and 100 *l.* at 10 months; at what time may the whole debt be paid together, without injury to either?

l.	*l.*	*l.*	*l.*
40	40	60	100
60	3	5	10
100	———	———	———
———	120	300	1000
200			300
			120
			———

2|00)14|20(

Answer, 7 months.

Note, The times of payment, if not ſo given, muſt be reduced into one name.

But thoſe who are more exact take the pains to work theſe ſums by the following rule.

Firſt, they find the preſent worth of every particular ſum; then find in what time to come the ſum of theſe preſent worths will be increaſed to the total of all the particular ſums payable at the ſeveral times to come; and that is the true equated time for payment of the whole.

CHAP.

CHAP. VIII.

Of FELLOWSHIP, or COMPANY.

FELLOWSHIP, or COMPANY, is when two or more join their stocks, and trade together, dividing their gain or loss proportionably.

FELLOWSHIP is either with, or without time. Questions without time, are worked by this proportion:

As the whole stock,
To the whole gain, or loss:
So is each man's particular stock,
To his particular share.

EXAMPLES.

A, *B*, and *C*, make a joint stock; *A* puts in 460 *l.* *B* 510 *l.* and *C* 480 *l.* they gain 340 *l.* what part of it belongs to each?

	l.
A,————	460
B,————	510
C,————	480
Total stock,	1450

1*st*. Then, *If* 145|0——340——46|0

```
         340
        ----
        1840
        138
        ---- l.
145)15640(107
     145
     ----
     .1140
      1015
     -----
     .125
       20
     ----- s.
145)2500(17
    145
    ----
    1050
    1015
    ----
    ..35
      12
    ---- d.
145)420(2
    290
    ---
    130
      4
    ---
145)520(3
    435
    ---
    .85
```

l. *l.* *l.*

2*d*. *If* 145|0—340—51|0

```
          340
         ----
         2040
         153
         ---- l.
145)17340(119
     145
     ----
     .284
      145
     ----
     1390
     1305
     ----
       85
       20
     ---- s.
145)1700(11
    145
    ----
    .250
     145
    ----
     105
      12
    ---- d.
145)1260(8
    1160
    ----
    .100
       4
    ----
145)400(2
    290
    ---
    110
```

3*d*.

l. *l.* *d.*
3*d.* And, *If* 145|0 ——— 340 ——— 48|0

```
           48|0
            340
           ----
           1920
           144
           ----  l.
     145)16320(112
           145
           ---
            182
            145
            ---
            370
            290
            ---
             80
             20
            ---  s.
      145)1600(11
            145
            ---
            150
            145
            ---
              5
             12
            ---
             60
              4
            ---
       145)240(1
            145
            ---
             95
```

	l.	*s.*	*d.*	*Rem.*
A's *Share is*	107 .	17 .	$2\frac{3}{4}$ —	85
B's *Share is*	119 .	11 .	$8\frac{1}{2}$ —	110
C's *Share is*	112 .	11 .	$0\frac{1}{4}$ —	95
Value of the Remainder			$\frac{1}{2}$ —	
				145)290(2
The whole gain	340 .	00 .	0 .	290
				.0

And thus are these Sums proved

More EXAMPPES.

Three persons make a joint stock; *A* puts in 565 *l.* *B* 278 *l.* and *C* 629 *l.* they gain 1000 *l.* and what is each of their shares?

		l.	*s.*	*d.*	*Remainders.*
Answer,	*A,*	383	16	$7\frac{1}{2}$	384
	B,	188	17	2	512
	C,	427	6	$2\frac{1}{4}$	576

D, *E*, and *F*, trade together; *D* puts in 230*l.* *E* 5?9*l.* and *F* 344 *l.* 10 *s.* their whole gain amounts to 520 *l.* what is that to each?

		l.	*s.*	*d.*	*Remainders.*
Answer,	*D,*	108	7	$7\frac{1}{4}$	271
	E,	249	5	7	844
	F,	162	6	9	1091

A, *B*, and *C*, make a bank of 3256 *l.* whereof *A* puts in 1026*l.* *B* 985 *l.* and *C.* the rest; by misfortune they lose 2000 *l.* what part of it must each bear?

		l.	*s.*	*d.*	*Remainders.*
Answer,	*A,*	630	4	5	928
	B,	605	0	$8\frac{1}{4}$	1242
	C,	764	14	10	1088

FELLOWSHIP *with* TIME.

FELLOWSHIP with TIME is thus worked: Multiply each man's stock by the respective time he puts it in for, and add all the products, the total of which must be your first number through all the statings; the gain,

gain, or loſs, the ſecond (as before ;) and each man's particular ſtock, multiplied by its time, the third.

Note, All the particnlar times (if not ſo given) muſt be reduced into one name, (*i. e.*) all years, all months, all weeks, or all days, *&c.*

The reaſon of multiplying the ſeveral ſtocks by the times they are put in for, will appear from the following conſiderations. *Firſt*, Suppoſe *A* and *B* put into trade each 100 *l.* for 1 month, then certainly their gain, whatever it be, muſt be equally divided betwixt them Again, ſuppoſe *A* puts in 100 *l*, and *B* 200 *l*. both for the ſame time; it is as plain that *B*'s ſhare of the gain muſt be twice as much as *A*'s, becauſe his ſtock is double: or if each put in 100 *l.* but for different times, *viz.* *A*, for one month, and *B*, for 2; then as *B*'s ſtock lies twice as long as *A*'s, ſo his gain, as before, muſt be twice as great. *Laſtly*, ſuppoſe *A* to put in 100 *l.* for one month, and *B* 200 *l.* for 2 months; then ſince his ſtock is not only double, but the time he leaves it in double too, his ſhare of the profits muſt become 4 times greater than *A*'s. And ſo in all other caſes.

EXAMPLE.

A put into company 560 *l.* for 8 months, *B* 279 *l.* for 10 months, and *C* 735 *l.* for 6 months; they gained 1000 *l.* what ſhare of it muſt each have?

l.	*l.*	*l.*
560	279	735
8	10	6
A 4480	*B* 2790	*C* 4410
		2790
		4480
		11680

1st. Then, If 1168|0 —— 1000 — 448|0 (l. l. l.)

```
                1000
                ------ l.
        1168)448000(383
              3504
              ----
              .9760
               9344
               ----
               .4160
                3504
                ----
                .656
                  20
                ----- s.
           1168)13120(11
                1168
                ----
                .1440
                 1168
                 ----
                 .272
                   12
                 ---- d.
            1168)3264(2
                 2336
                 ----
                 .928
                    4
                 ----
            1168)3712(3
                 3504
                 ----
                 .208
```

2d.

l. l. l.
2d. If 1168|0——1000——279|0

```
                  1000
                  ——— l.
      1168)279000(238
           2336
           ————
           .4540
            3504
           ————
           10360
            9344
           ————
            1016
              20
           ———— s.
      1168)20320(17
           1168
           ————
           .8640
            8176
           ————
            .464
              12
           ———— d.
      1168)5568(4
           4672
           ————
            .896
               4
           ————
      1168)3584(3
           3504
           ————
           ..80
```

	l.		*l.*		*l.*
3*d.* *If*	1168|0	——	1000	——	441|0

	l.	*s.*	*d.*	*Rem.*
A's Share is	383	11	$2\frac{3}{4}$	208
B's Share is	238	17	$4\frac{3}{4}$	80
C's Share is	377	11	$4\frac{1}{4}$	880
Value of the Remainders			$\frac{1}{4}$	1168)1168(1
	1000	00	0	...0
The whole gain.				

```
        441|0
         1000
        ------ l.
1168)441000(377
     3504
     ----
     9060
     8176
     ----
     8840
     8176
     ----
      664
       20
     ----- s.
1168)13280(11
     1168
     ----
     1600
     1168
     ----
      432
       12
     ---- d.
1168)5184(4
     4672
     ----
      512
        4
     ----
1168)2048(1
     1168
     ----
      880
```

More

More EXAMPLES.

A, *B*, and *C*, agreeing to trade together, *A* puts in 529 *l*. for 4 months; *B* 329 *l*. for 7 months; and *C* 900 *l* for 2 months; by their trade they gain 540 *l*. What part of it belongs to each?

		l.	*s.*	*d.*	*Remainders.*
Answer,	*A,*	183	14	8	—— 2304
	B,	199	19	5	—— 1222
	C,	156	5	10¾	—— 2583

Theee persons make a joint stock; *A* puts in 400 pieces of holland, each containing 96 yards, at 5*s*. 6*d*. *per* ell *Flemish* for 5 months; *B* 600 guineas for 8 months; and *C* 1800 *l*. for 4 months, but at 3 months end takes out 800 *l*. They gain in all 1000 *l*. how much is each of their shares?

		l.	*s.*	*d.*	*Remainders.*
Answer,	*A,*	860	4	3½	—— 3996
	B,	61	11	8	—— 1920
	C,	78	4	0¼	—— 2568

CHAP.

CHAP. IX.

Of ALLIGATION.

ALLIGATION is the compounding many simples into one mass, according to any required price, or proportion.

This rule is usually divided into two parts, distinguished by the names of *Medial* and *Alternate*.

ALLIGATION MEDIAL.

ALligation Medial answers such questions as, having the quantities and prices of the several simples given, only demand the mean rate of any particular part of the composition.

RULE.

As the sum of all the simples,
To their total value :
So is any quantity of the mixture,
To the price thereof.

EXAMPLES.

(*Ex. 1st.*) A grocer would mix 18 lb. of currants at 5*d. per* lb. and 29 lb. at 6*d. per* lb. with 12 lb. at 8 *d. per* lb. at what rate may he sell a pound of the mixture ?

First,

First, place the quantities and their values, as below.

℔. d.		s. d.
18 at 5 *per* ℔. *comes to*		7 . 6
29 at 6 ——— *to*		14 . 6
12 at 8 ——— *to*		8 . 0
Sum of all the Sim. 59		£. 1 . 10 . 0 *tot. val.*

Then, *If* 59 *comes to* 1 *l.* . 10 *s. what will* 1 ?

20

—

30

12

— *d.*

59)360(6 *Answer*.

354

—

6

Ex. 2. A grocer would with 12 *C.* of sugar, at 3 *l. per C.* mix 3 *C.* at 50 *s. per C.* and 8 *C.* at 45 *s. per C.* What will 5 *C.* of this mixture be worth?

The quantities and their values, placed as before, will stand thus:

C.		l. s.
12 *at* 3 *l. per C. is*		36 . 00
3 *at* 50 *s. per C. is*		7 . 10
8 *at* 45 *s. per C. is*		18 . 00
Sum of the Simples 23		61 . 10 *tot. val.*

Then,

```
           C.          l.   s.        C.
Then, If 23 ———————— 61 . 10 ———————— 5
                       20
                      ————
                      1230
                         5
                      ————
                   23)6150(267
                      46   ———
                  ——————— 13 l. 7 s. 4 d. ½ facit.
                      155
                      138
                      ———
                       170
                       161
                       ———
                         9
                        12
                       ———
                    23)108(4
                        92
                       ———
                        16
                         4
                       ———
                     23)64(2
                        46
                       ———
                        18
```

The proof of this is as in the *Rule of Three*, only by reversing the proportion.

So the first example may be proved by this back-stating

```
        lb.          d.          lb.
      If 1 ———————— 6 ———————— 59
```

And the last by this;

```
        C.     l.   s.  d.     C.
      If 5 ——— 13 . 7 . 4½ ——— 23
```

remembering in both to take in the remainders.

More

More EXAMPLES.

If a vintner mingleth 13 gallons of canary, at 6*s*. 8*d*. *per* gallon, with 20 gallons of white-wine, at 5*s*. *per* gallon, and 10 gallons of cyder, at 3*s*. *per* gallon, at what rate must he sell a quart of the mixture?

Answer, 1*s*. 3*d*.

If with 40 bushels of corn, at 4*s*. *per* bushel, there are mixed 10 bushels at 6*s*. *per* bushel; 30 bushels at 5*s*. *per* bushel, and 20 bushels at 3*s*. *per* bushel, what will 10 bushels of the mistling be worth?

Answer, 2*l*. 3*s*.

ALLIGATION ALTERNATE.

ALligation Alternate teacheth what quantities of several simples (whose values are given) must be mixed, that the composition may bear a price propounded.

This sort of *Alligation* has three varieties.

VARIETY 1.

When the price of each simple is expressed, but no quantity given, and it is required how much of each simple must be mixed, to sell any part of the composition at a mean price propounded?

In this variety you have only to link the several extremes rightly together, and to find the true differences betwixt them and the mean; for those differences are the quantities sought.

EXAMPLE.

A vinter would mix four sorts of wine, of 12 *d*. of 18 *d*. of 24*d*. and of 26*d*. *per* quart; what quantity of each must he take to sell the mixture at 20*d*. *per* quart?

To

To place your numbers, and link the extremes, observe these

RULES.

1*st*. Set down the ſeveral given prices (which muſt always be of one name) in a column, one under another; then, towards your left hand, draw a line of connection; and on the other ſide of the ſaid line place the mean rate.

Thus then will ſtand the given numbers of the given example:

$$20\begin{cases}12\\18\\24\\26\end{cases}$$

2*d*. Link them two and two together, always obſerving to join a greater and a leſs than the mean; and againſt each extreme place the difference betwixt the mean and its yoke-fellow:

Thus,

$$20\left\{\begin{array}{ll}12 & 4\\18 & 6\\24 & 8\\26 & 2\end{array}\right.$$

So the difference betwixt the extreme 12, and 20 the mean, being 8, is placed againſt 24, its yoke-fellow; and between 18 and 20 being 2, is placed againſt 26: ſo alſo the difference betwixt 24 and 20 being 4, is placed againſt 12; and 6, the difference betwixt 26 and 20, is ſet againſt 18.

Therefore, if he mixes 4 quarts at 12 *d*. 6 at 18 *d*. 8 at 24 *d*. and 2 at 26 *d*. *per* quart, he may ſell the mixture at 20 *d*. *per* quart.

The proof of which is eaſy. For the ſum of the differences valued at the mean rate, will be found equal to the amount of the particular differences at their given prices.

For

For 20 quarts, the sum of the differences at 20 *d. per* quart, is 400 pence; and 4, 6, 8, 2 quarts, the particular differences, at 12, 18, 24, 26 pence *per* quart, will come to the same sum.

But note, as many different ways as the extremes may be combined, so many different answers may be given to the question, yet all true.

So the same example being combined, thus:

$$20\left\{\begin{array}{l}12\\18\\24\\26\end{array}\right.\left|\begin{array}{l}6\\4\\2\\8\end{array}\right\}\textit{Answ.}\qquad 20.\left\{\begin{array}{l}12\\18\\24\\26\end{array}\right.\left|\begin{array}{l|l}4.6&10\\4.6&10\\8.2&10\\8.2&10\end{array}\right\}\textit{Answ.}$$

gives those different, but true answers, as may be proved by the same rule.

In the last of these examples the numbers being doubly combined (*i. e.*) each with two others, make each have two differences set against it; for 12, the first extreme, being linked both with 24 and 26, must have both their differences, *viz.* 4 and 6, placed against it; and so the rest. Which double difference must be added, and placed as above.

But when in a question there is but one extreme less, or but one greater than the mean; such a question will admit of but one answer, that single extreme being to be linked with all the rest.

EXAMPLE.

A grocer would mix sugars, at 5 *d.* 6 *d.* and 10 *d. per* ℔. so as to sell the compound at 8 *d. per* ℔. what quantity of each must he take?

$$8\left\{\begin{array}{c}5\\6\\10\end{array}\right.\left|\begin{array}{l}2\\2\\3.2\end{array}\right|\begin{array}{l}2\\2\\5\end{array}$$

The numbers being placed, linked, and differenced, (as above) according to the foregoing rules, it appears, to sell his sugar at 8 *d. per* ℔. he must mix 2 ℔. at 5 *d.* 2 ℔. at 6 *d.* and 5 ℔. at 10 *d. per* ℔.

The

The reason of these combinations, and the alternate placing of their differences, will appear from this plain consideration, *viz.* that thereby whatever is lost upon the quantity sold, whose given price exceeds the mean, is gained again upon the quantity, whose given price is less than the mean.

More EXAMPLES.

A tobacconist wou'd mix tobacco of 2*s.* and 1*s.* 6*d.* and 1*s.* 3*d.* *per* ℔. so as to sell the compound at 1*s.* 8*d.* *per* ℔. what quantity of each must he take?

	℔.		*s.*	*d.*	
Answer,	7	at	2	0	*per* ℔.
	4	at	1	6	
	4	at	1	3	

Several sorts of tea, *viz.* of 13*s.* 18*s.* 26*s.* and 30*s.* *per* ℔. are to be so mixed, that the compound may be worth 22*s.* *per* ℔. what quantity of each must there be?

	℔.		*s.*	
Answer,	8	at	13	*per* ℔.
	4	at	18	
	4	at	26	
	9	at	30	

VARIETY 2.

Here we have the price of all the simples, and the quantity of one given, to find the several quantities of the rest: so that one measure or quantity, may be sold at a mean rate propounded.

When the numbers have been set down, and their several differences found by the foregoing rule, you must proceed by this proportion.

As the difference of that number whose quantity is given,

To the rest of the differences one after another:

So the quantity given,

To the several quantities sought.

EXAM-

E X A M P L E.

A distiller would with 40 gallons of *French* brandy, at 12*s.* *per* gallon, mix *English* brandy at 7*s.* *per* gallon, and spirits at 4*s.* *per* gallon: how much of the last two sorts must be added, to sell the whole at 8*s.* *per* gallon?

8	12	1 . 4	5
	7	4 .	4
	4	4 .	4

Now, it is plain, that were there but 5 gallons of the *French* brandy, there must also be but 4 gallons of each of the other liquors; but since there are to be 40 gallons of the *French* brandy, say,

As 5, the difference against its price,

To 4, the difference against the price of the *English* brandy,

So is 40, the given quantity of the *French*,

To the quantity of the *English* sought:

Which by this proportion should be found to be 32.

Then again,

As the first difference 5,

To 4, the difference against the price of the spirits;

So 40, the said given quantity,

To the quantity of spirits sought.

Which because their differences are alike, will be the same as of the *English* brandy, *viz.* 32.

This variety has the same proof as the former.

More E X A M P L E S.

How much tea at 14*s.* *per* ℔. and of 18*s.* *per* ℔. must be mixed with 8 ℔. at 24*s.* *per* ℔. so as to allow the ℔. at 20*s.*?

Answer, { 8 ℔. at 24*s.*
4 ℔. at 18*s.*
4 ℔. at 14*s.*

I would with 21 ℔. of snuff, at 8*s.* *per* ℔. mix so much of 4*s.* *per* ℔. and so much of 5*s.* *per* ℔. that I might

might afford to sell the mixture at 6 *s. per* lb. what quantity of each must I take?

Answer, $\left\{\begin{array}{l} 21 \text{ at } 8 \\ 14 \text{ at } 4 \\ 14 \text{ at } 5 \end{array}\right\}$ *per* lb.

VARIETY 3.

In this variety the particular prices of each simple being given, as also the mean rate of the compound, it is required how much of each sort must be taken, to make up the quantity propounded?

Which (after the work of the first variety) is performed by this proportion;

As the sum of the differences.
To the total quantity:
So each particular difference,
To its particular quantity.

EXAMPLE.

A druggist hath 4 sorts of green tea, at 5*s.* 6*s.* 8*s.* and 9*s. per* lb. and would make up a quantity of 87 lb. to sell at 7*s. per* lb. how much of each must he take?

See the work.

			lb.	*s.*
7	5	2	29	*at* 5 *per* lb.
	6	1	$14\frac{1}{2}$	*at* 6 *per* lb.
	8	1	$14\frac{1}{2}$	*at* 8 *per* lb.
	9	2	29	*at* 9 *per* lb.
		6		

lb. lb. lb.
If 6 —— 87 —— 2
2
—— lb.
6)174(29
——
0

lb. lb. lb.
If 6 —— 87 —— 1
1
—— lb.
6)87($14\frac{1}{2}$
——
3

More

More EXAMPLES.

A vintner hath 3 sorts of wine, of 6*s*. 8*s*. and 10*s*. *per* gallon, of all which he would compose a quantity of 120 gallons, to sell at 7*s*. *per* gallon: what quantity must there be of each?

$$\textit{Answer,}\left\{\begin{array}{l} \textit{gall.}\quad s. \\ 80 \textit{ at } 06 \\ 20 \textit{ at } 08 \\ 20 \textit{ at } 10 \end{array}\right\} \textit{per } \text{Gall.}$$

A grocer having sugars of 5*d*. 6*d*. 8*d*. 10*d*. and 11 *d*. *per* lb. would make a mixture of 3 *C. wt*. to sell at 7*d*. *per* pound: how much of each must he take?

$$\textit{Answer,}\left\{\begin{array}{l} \text{lb.}\quad d. \\ 28 \textit{ at } 05 \\ 196 \textit{ at } 06 \\ 56 \textit{ at } 08 \\ 28 \textit{ at } 10 \\ 28 \textit{ at } 11 \end{array}\right\} \textit{per } \text{lb.}$$

CHAP.

CHAP. X.

Of POSITION, *or the Rule of* FALSE.

THIS rule hath not the name of *False*, from its being in itself really erroneous, but only because that by the help of false supposed numbers we find the truth.

It is usually divided into two parts, *single* and *double*.

Single POSITION.

IN *Single Position* we make but one supposition, with which working as with a true number, we regulate the result by this proportion, *viz.*

As the total arising from the error,
To the true total:
So the supposed part,
To the true one.

EXAMPLES.

1*st*. *A*, *B*, and *C*, designing to buy a quantity of lead, to the value of 140 *l.* agree that *B* shall pay as much again as *A*, and *C* as much again as *B*, what then must each pay?

Now

Now suppose *A* to pay 10 *l.* then *B* must pay 20 *l.* and *C* 40 *l.* the total of which is 70 *l.* but should be 140 *l.* Therefore,

If 70l. should be 140 *l.* what should 10 *l. be?*

l.

Answer, 20 *for* A's *share;*
which doubled, makes 40 *for* B's *share;*
and that again doubled, gives 80 *for* C's *share;*

The total of which is 140

2. Suppose the sum of 20 *l.* were to be paid by *A*, *B*, and *C*, thus, $A\frac{1}{2}$, $B\frac{1}{3}$, $C\frac{1}{4}$; what must each pay according to that proportion;

	l.	s.	d.
First, the $\frac{1}{2}$ *of* 20*l. being*	10	00	0
the $\frac{1}{3}$ ———	6	13	4
and the $\frac{1}{4}$ ———	5	00	0
The total will be	21	13	4

But it should have been but 20*l. therefore the parts found should be thus regulated:*

l. s. d.	l.	l. s. d.		l. s. d.
If 21 13 4 *fall to* 20,		10 . 00 . 0	*Answer*	9 . 04 . $7\frac{1}{2}$
what will		06 . 13 . 4		6 . 03 . $0\frac{3}{4}$
		05 . 00 . 0		4 . 12 . $3\frac{3}{4}$

which, with the addition of $\frac{1}{2}$, *makes just* 20 . 00 . 0

found by dividing the total of the remainders ——— {28, 36, 40}

by the common divisor 52) 104 (2

0

More EXAMPLES.

A gentleman bought a chaise, horse, and harness, for 50 *l.* the horse come to twice the price of the harness,

3

harneſs, and the chaiſe to twice as much as both horſe and harneſs; what did he give for each?

		l.	s.	d.	Remainders.
Anſwer,	the harneſs	05	11	$1\frac{1}{4}$	—— 15
	the horſe	11	02	$2\frac{1}{2}$	—— 30
	the chaiſe	33	06	8	——

A, *B*, and *C*, diſputing of their age, *A* affirms he is as old as *B*, and $\frac{1}{2}$ as old as *C*, and *B* ſays, that he is $\frac{3}{4}$ as old as *C*, upon which *C* ſays, he is ſure both their ages, added to his, will make 110 years. I demand the age of each.

Anſwer { *A*, $45\frac{5}{6}$; *B*, $27\frac{3}{6}$; *C*, $36\frac{4}{6}$ }

Double Position.

IN *Double* Position two ſuppoſitions are required, which (if both prove falſe) are, with their errors, to be thus diſpoſed.

Set down both your ſuppoſitions, and againſt each of them their reſpective errors, marked thus +, if too much; or thus —, if too little.

Then multiplying them croſs-ways, that is, the firſt ſuppoſition by the ſecond error, and the ſecond ſuppoſition by the firſt error, if both the errors are alike, *i. e.* both too little, or both too much, ſubtract the leſſer product from the greater, and divide the remainder by the difference of the errors. But if the errors are unlike, *i. e.* one too little, and the other too much, the ſum of theſe products muſt be divided by the ſum of the errors: the quotient will give the true number, exactly anſwering all the demands of the queſtion.

Or the ſuppoſitions and their errors being placed as before, work by this proportion as a general rule, *viz.*

As the difference of the errors, if alike,
Or their ſum, if unlike,
To the difference of the ſuppoſitions :
So either error to a fourth number,
which accordingly added to, or ſubtracted from the ſuppoſition againſt it, will anſwer the queſtion.

EXAMPLES.

1*ſt*. *A*, *B*, and *C*, playing at cards, the money ſtak'd was 324 crowns ; but diſagreeing, each ſeized as many as he could ; *A* got a certain number, *B* as many as *A*, and 15 over ; *C* got a fifth part of both their ſums added together : how many had each ?

First, *ſuppoſe* A *got* 50
then B'*s ſhare will be* 65
and C'*s* ——— 23

All which added, make but 138, *which is* 186 *too few.*
Then, *ſuppoſing again* that A *got* 80
B'*s ſhare will be* 95
and C's ——— 35

The total of which is ſtill but 210, *too few by* 114.

Therefore, ſetting down the ſuppoſitions, and their errors (mark'd as before directed) againſt them, they will ſtand thus:

Suppoſitions.	*Errors.*
50	186
80	114

then multiplied croſs-ways, the products will be 14880, and 5700 ; the leſſer of which (the errors being alike, *i. e.* both too little) being taken from the greater, there will remain 9180 for a dividend ; which divided by 72, the difference of the two errors, qotes $127\frac{1}{2}$, the true number of crowns *A* took up ; as will appear, by adding 15 to them for *B's* ſhare, and dividing the ſum of *A's* and *B's* by 5,

F for

for *C's* share; for so their particular number will be found to be as follow:

A's———$127\frac{1}{2}$
B's———$142\frac{1}{2}$
C's——— 54

The total of which is 324, *the number of crowns given.*

Or by the other rule;

As 72, the difference of the errors (because alike) to 30, the difference of the suppositions:

$$\text{So is} \left\{\begin{matrix} 186 \\ or \\ 114 \end{matrix}\right\} to \left\{\begin{matrix} 77\frac{1}{2} \\ \\ 47\frac{1}{2} \end{matrix}\right\} \begin{matrix} which \\ added \\ to \end{matrix} \left\{\begin{matrix} 50 \\ or \\ 80 \end{matrix}\right\} \text{gives } 127\tfrac{1}{2}, \text{ as before.}$$

EXAMPLE 2.

When first the marriage-knot was ty'd
Betwixt my wife and me,
My age did her's as far exceed,
As three times three does three.
But after ten and half ten years,
We man and wife had been,
Her age came up as near to mine,
As eight is to sixteen.
———————————Now, pray,
What were our ages on the wedding-day?

First, *suppose her* 21 *then he must be* 63
then adding to each 15 ———————— 15

her age becomes 36 *his* 78

This error therefore is 3 years too few.

But supposing her 9, the error will prove 3 too many.

Setting down therefore against the suppositions the errors, with their respective marks, they will stand thus:

Sup-

Suppositions.		*Errors.*
21	—	3
9	+	3

Then multiplying cross-ways, 3 times 21 is 63
3 times 9 is 27

These products (because unlike) added, make 90 for a dividend, which divided by 6, the sum of the errors, quotes 15, her true age; proved thus:

She being 15, *he must be* ——— 45
15 *years added to each* 15

Make her 30, *just half of his* —— 60

Or, by the other rule:

As 6, the sum of the errors, (because unlike) to 12, the difference of the suppositions:

So is the common error, 3 to 6; which either subtracted from 21, or added to 9, gives 15, as before) for her age.

More EXAMPLES.

What number is that to which if you add $\frac{1}{4}$ of itself, and from the sum subtract $\frac{1}{3}$ of itself, the remainder will be 210?

Answer, 229.

A gentleman caught a fish, whose head was 6 inches long, the tail as long as the head, and half as long as the body; the body was just the length of the head and tail: I demand the length of the fish?

Answer, 48 inches, or 4 foot.

CHAP. XI.

Of EXCHANGE.

EXCHANGE is the receiving money in one country, for the value paid in another.

NOTE.

The par of exchange is certain and fix'd, being always *par pro pari, like for like,* according to the weight and fineneſs of the coin.

But the courſe or current running price betwixt any two countries, riſes and falls upon every occaſion.

It would be both needleſs and endleſs to write of every kind of exchange; I ſhall therefore only give examples of the exchange of *England,* with ſome other chief countries of *Europe.* And firſt, with

FRANCE.

At *Paris, Lyons, Rouen,* &c. they keep their accounts in livres, ſols, and deniers; and exchange upon the crown, the par of which, in ſterling money is 4*s.* 6*d.*

NOTE.

NOTE.

12 Deniers	make	1 Sol.
20 Sols		1 Livre.
3 Livres		1 Crown.

French money is changed into *Sterling* by this proportion;

As a Crown to the given rate:
So is the given *French* sum to the *Sterling* sought.

Or,

By supposing the given *French* crowns, *goods*, and the rate of exchange, the *price*; so casting up their value by *practice*.

EXAMPLES.

1*st*. What *sterling* money must be paid in *London*, to receive in *Paris* 438 crowns; exchange at 56 *d.* *per* Crown?

Cr. d. Cr.
If 1——56——438
56
2628
2190
12)24528(
2|0)204|4
102 *l.* 4 *s.*

Cr. d. s. d.
438 at 56 or 4 8 *per Cr.*
4 *s.* $\frac{1}{3}$ | 87 . 12
8 *d.* $\frac{1}{6}$ | 14 . 12
102 . 4

2*d.* Change 537 crowns, 14 sols, 10 deniers, into *Sterling* money; exchange at 52 *d.*½ *per* crown.

Cr. *d.* *Cr.* *Sols.* *De.*

If 1—52½—537 . 14 . 10
60 2 60
12—— ——
——105 32234
720 12
——
386818
105
——
1934090
3868180
—— 2
72|0)4061589|0(56410
360 ——
—— 12)28205(5
.461 ——
432 2|0)235|0
—— ——
.295 117*l.* 10*s.* 5*d.*¼ *facit.*
288
——
.78
72
——
69
2
——
72)138(1
72
——
66

Cr. *Sols.* *De.* *d.*
537 . 14 . 10 at 52½
——
4*s.* 1/5 107 . 8
4*d.* 1/12 .8 . 19
½ 1/8 .1 . 2 . 4½
Sols 10 1/6 . . . . . 8¾
2 1/5 . . . . . 1¾
2 1/5 . . . . . 1¾
Den. 10 1/12 *of* 10 *Sols.* ½
——
117 *l.* 10*s.* 5*d.*¼ *fac.*

Sterling

Sterling money is changed into *French* by this proportion:

As the rate of exchange, to one crown:
So is the given *English* sum, to the *French* sought.

For EXAMPLES.

Take the reverse of the former questions; which will also be their proof.

1*st*. How many *French* crowns must be paid in *Paris*, to receive in *London* 102*l.* 4*s.* *Sterling*; exchange at 56 *d.* *per* crown?

```
     d.        Cr.        l.    s.
  If 56 ——— 1 ——— 102 . 4
                       20
                     ————
                     2044
                       12
                   ————Crowns.
                56)24528(438
                   224
                   ———
                    212
                    168
                    ———
                     448
                     448
                     ———
                       0
```

2d. Change 117*l.* 10*s.* 5*d.*¼ *Sterling* into *French* crowns; exchange at 52 *d.*½ *per* crown.

```
     d.      Crown.     l.     s.    d.
If 52½ ——— 1 ——— 117 . 10 . 5¼
     4                  20
   ———                ———
   210                2350
                        12
                     ————
                     28205
                         4
  .51              ———————Crowns Sols. Deniers.
   60        21|0)11282|1(537 .  14 . 10
  ——— Sols.        105
21.0)306|0(14      ———
   21               .78
  ———                63
  .96               ———
   84                152
  ———                147
   12                ———
   12                .51
  ———
  144
   66 the remainder of the other sum.
  ———
 21)210(10 deniers.
   21
   —
   0
```

More EXAMPLES.

If I pay in *Paris* 8297 *crowns*, 13 *sols*, 8 *deniers*, what may I draw my bill for to *London*, when the exchange is at 54*d.*¼ *per* crown?

Answer, 1875*l.* 10*s.* 4*d.*½.

What *Sterling* money is equal to 954 *crowns*, 52 *sols*, when the exchange is at 58*d.*⅛ *per* crown?

Answer, 231*l.* 5*s.* 1*d.*½.

If I pay in *London* 536*l.* what *French* money shall I receive in *Paris* exchange at 57 *d. per* crown?

Answer, 2256 *crowns*, 50 *sols*, 6 *deniers*.

What *French* money is equal to 1529 *l.* 10 *s.* exchange at 53 *d.* ¼ *per* crown?

Answer, 6893 *crowns*, 31 *sols*, 3 *deniers*.

Secondly, with ITALY.

IN *Genoa*, *Leghorn*, &c. they keep their accompts in livres, sols, and deniers; and exchange upon the piece of eight, or dollar; the *par* of which with *London*, is also 4*s.* 6*d.* *Sterling*.

NOTE.

At *Genoa* 5 livres make a piece of eight, at *Leghorn* 6. Observe the rules for exchanging with *France*.

EXAMPLES.

How much *Sterling* shall I receive in *London*, if I pay in *Genoa* 820 dollars; exchange at 51*d. per* dollar, or piece of eight.

Dollar.	*d.*	*Dollars.*		*Dollars.*
If 1 ——	51 ——	820		820 at 51*d. per* dollar.
		51		4
		820	*d.*	3280
		4100	3¼	205
		12)41820(		348|5
		2|0)348|5		174*l.* 5*s.*
		174*l.* 5*s.*		

Or

Or back again;

If in *London* I pay 174*l.* 5*s.* *Sterling*, how many dollars shall I receive in *Genoa*; exchange at 51*d.* *per* dollar?

	d.	*Dol.*	*l.*	*s.*
If	51	1	174	5

```
              20
             ----
             3485
              12
             ----
      51)41820(820
         408
         ---
         .102
          102
          ---
          ...0
```

Venice exchanges by the ducat; the *par* about 56 *d.* $\frac{1}{4}$ *Sterling.*

NOTE.

6 Solidi } make { 1 Gross.
24 Grosses } make { 1 Ducat.

EXAMPLE.

Change 512 ducats into *Sterling* money; exchange at 55*d.*$\frac{1}{2}$ *per* ducat.

```
   D.     d.       D.                Ducats.   d.
If 1 —— 55½ —— 512*                   512 at 55½ per Duc.
         2       512                    4
        ---      512                  ----
        111   ---------               2048
            2)56832(    d.             256
            ---------   6½              64
           12)28416(    1½ ¼          ----
            ---------                 2368
            2|0)2368                  ----
            ---------                 118l. 8s.
              118l. 8s.
```

* See Abbrevations of Multiplication.

Thirdly,

Thirdly, with SPAIN.

IN *Madrid, Seville, &c.* they keep their accompts in rials and mervadies, and exchange also by the piece of eight. The *par* with *London* 4*s*. 6*d*.

NOTE.

34 Mervadies	make	1 Rial.
8 Rials		1 Piece of $\frac{8}{8}$.

This exchange is also cast up by the foregoing rules.

EXAMPLE.

What *Sterling* money may I draw for to *London*, if I pay in *Seville* 4798 rials, 25 mervadies; exchange at 58 *d. Sterling per* piece of $\frac{8}{8}$?

```
Rials.   d.    Rials.  Mex.
If 8 —— 58 —— 4798 . 25
   34            34
  ———          ———
  272          19197
              14396
              ———
              163157
                  58
             ————
             1305256
             815785
            ———— 12
       272)9463106(34790(2
            816        ———
          ———— 2|0)289|9(
            1303       ———
            1088         144 . 19 . 2¾
            ———
            2151
            1904
            ———
            2470
            2448
            ———
             ..226
                  4
              ———
          272)904(3
              816
              ———
               .88
```

Or, back again.

Change 144*l*. 19*s*. 2*d*.¼ *Sterling* into *Spanish* money; exchange at 58*d*. *per* piece of $\frac{8}{8}$.

```
   d.   Rials.   l.    s.   d.
If 58———8———144 . 19 . 2¼
    4          20
   ———        ————
   232        2899
                12
              ————
             34790
                 4
              ————
            139163
                 8
              ————
      232)1113304(4798 Rials.
              928
              ————
              1853
              1624
              ————
              2290
              2088
              ————
              2024
              1856
              ————
               168
                34
              ————
               672
               504
                88 Rem. of the other.
              ————
      232)5800(25 Mervadies.
              464
              ————
              1160
              1160
              ————
              ...0
```

More

More ***EXAMPLES.***

Change 83927 *Rials* into pounds *Sterling*; exchange at 54*d*.½ *per* piece of eight.

Answer, 2382*l*. 6*s*. 0*d*.½.

If I pay in *London* 5000*l*. what *Spanish* money may I draw my bill for to *Madrid*; exchange at 57 *d*. ⅛ *per* piece of eight?

Answer, 168052 *Rials*, 17 *Mervadies*.

Fourthly, with PORTUGAL.

IN *Lisbon*, *Oporto*, *&c.* they keep their accompts in Rees, and exchange on the Mill-ree; the *par* of which is about 6*s*. 8*d*.½ *Sterling*.

NOTE.

1000 Rees make 1 Mill-ree.

To make this exchange still observe the same rule.

EXAMPLES.

Change 48009 rees into Sterling money; exchange at 6 *s*. 5*d*. *per* mill-ree.

```
 MR      s.  d.      Rees.
 If 1 —— 6 . 5 —— 48009
 1000    12             77
         ——         ———————
         77          336063
                    336063
                   ———————— 12
           1|000)3696|693(3696)
                           4 ————————
                  ————————2|0)30|8
                   2|772 ————————
                          15l. 8s. 0d.½
```

Or,

Or, back again,

Change 15 *l.* 8*s.* 0*d.*$\frac{1}{2}$ *Sterling* into rees; exchange at 6*s.* 5*d.* *per* mill-ree.

```
  s.   d.         Rees.        l.   s.   d.
If 6 . 5 ————— 1000 ————— 15 . 8 . 0½
  12                          20
 ———                         ———
   77                         308
    4                          12
 ———                         ———
  308                         3696
                                 4
                              ———
                             14786
                               1000
                       ————————————— Rees.
Note, 772, Rem. of   308)14786772(48009
the other stating is      1232
here taken in.            ————
                          2466
                          2464
                          ———————
                             2772
                             2772
                             ————
                             ...0
```

More EXAMPLES.

What Sterling money is equal to 595677 rees; exchange at 6*s.* 7*d.* *per* mill-ree?

Answer, 196*l.* 1*s.* 6*d.*$\frac{1}{4}$.

Change 59*l.* 12*s.* into rees; exchange at 6*s.* 10*d.* $\frac{1}{2}$ *per* mill-ree.

Answer, 173381 rees.

Fifthly,

I

Fifthly, HOLLAND, FLANDERS, *and* GERMANY.

I Take these places together, because their accompts are kept, and their exchange with *England* made the same way.

For not only in *Amsterdam*, but also in *Antwerp* and *Hamburgh*, they keep their accompts in pounds, shillings, and pence *Flemish*; or guilders, stivers, and pennicks; and exchange with us upon our pound, giving us for it, when at *par*, 33*s*. 4*d*. *Flemish*.

NOTE.

Their pounds, shillings, and pence, are divided as ours; *viz.* their pound into 20 shillings, and their shilling into 12 pence.

Their other money is thus divided.

16 Pennicks } make { 1 Stiver.
20 Stivers } make { 1 Guilder.

Note also,

6 Stivers } make { 1 Shilling } *Flemish*.
6 Guilders } make { 1 Pound } *Flemish*.

Sterling money is changed into *Flemish* by this proportion; *viz.*

As 1 *Sterling* to the given rate:
So is the given *Sterling* to the *Flemish* sought.

Or,

By supposing the given Sterling, *goods*, and the given rate their *price*, casting up their value by *practice*.

EXAM-

EXAMPLE.

If I pay in *London* 492 *l.* Sterling, what may I draw my bill for to *Amsterdam*; exchange at 34*s.* 5*d.* *Flemish per* pound Sterling?

	l.	*s.* *d.*	*d.*
If	1	34 . 5	492
		12	413
		413	1476
			492
			1968
			12)203196(
			2|0)1693|3(
			846*l.* 13*s.* *Fl.*

Or by Practice,

		l. *s.* *d.*
		492 at 34 . 5 *per l.*
s	10 $\frac{1}{2}$	246
	4 $\frac{1}{5}$	98 . 8
d.	4 $\frac{1}{12}$	8 . 4
	1 $\frac{1}{4}$	2 . 1
		846*l.* 13*s.* *facit.*

If you would have the answer in guilders and stivers, Divide the *Flemish* pence by 40, (the number of pence which make a guilder) and the quotient will be guilders, and if any thing remains, the half of it will be stivers, two *Flemish* pence being one stiver.

Or,

Multiply the *Flemish* pounds and shillings by 6, and it will produce guilders and stivers; for 6 guilders make 1 pound *Flemish*; and 6 stivers 1 shilling *Flemish*; and if there be any odd pence, multiply them by 8, for pennicks.

EXAMPLE.

846 *l.* 13 *s.* *the facit of the last sum.*
6

5079 *G.* 18 *S.*

Flemish pence.
4|0)20319|6(36 *Rem.*

5079 *Guild.* 18 *Stivers.*

Flemish

Flemiſh money (whether pounds, ſhillings, &c. or guilders, ſtivers, and pennicks) is exchanged into Sterling thus.

As the given rate to one pound Sterling;
So is the given *Flemiſh* to the Sterling ſought.

EXAMPLES.

Change 846 *l.* 13 *s.* *Flemiſh* into Sterling money, exchange at 34 *s.* 5 *d.* *Flemiſh per l.* Sterling.

```
   s.  d. Flem.  l. Sterl.   l.    s. Flem.
If 34  5 ——————— 1 ——————— 846   13
   12                       20
   ——                     —————
   413                    16933
                             12
                          ——————
                    413)203196(492 l. Sterling.
                        1652
                        ————
                         3799
                         3717
                         ————
                           826
                           826
                           ———
                             0
```

Or,

Change 5079 guilders, 18 ſtivers, into Sterling money; exchange 34 *s.* 5 *d. per l.* Sterling.

If

	s.	d. Flem.	l. Sterl.	Guild.	Stiv.
If	34	5 ——	1 ——	5079	18
	12			20	
	413			101598	
				2	

413)203196(492*l. Sterl.*
1652

3799
3717

826
826

0

More EXAMPLES.

Change 475 *l.* Sterling into *Flemish* money; exchange at 33*s. Flemish per l.* Sterling.

Answer, 783*l.* 15*s. Flemish.*

What *Flemish* money is equal to 365 *l.* 12 *s.* exchange at 36 *s.* 5*d. per l.* Sterling?

Answer, 665*l.* 13*s.* 11 *d. Flemish.*

If I pay at *Amsterdam* 5389 *l.* 14 *s. Flemish*, what Sterling money shall I receive at *London*; exchange at 32 *s.* 8*d. Flemish per l* Sterling?

Answer, 3299 *l.* 16 *s.* 3*d.*$\frac{3}{4}$.

Change 572 *l.* 13 *s.* Sterling into guilders, stivers, &c. exchange at 34*s.* 10*d. Flemish per l.* Sterling.

Answer, 5984 *guilders*, 3 *stivers*, 13 *pennicks*.

What may I draw my bill for to *London*, if I pay in *Antwerp* 7256 *guilders*; exchange at 35*s.* 2 *d.*$\frac{1}{2}$ *per l.* Sterling?

Answer, 686 *l.* 19 *s.* 2*d.* Sterling.

An

An Explanation of ſome Characters, ſometimes uſed for Brevity's ſake.

+ IS the mark of *Addition*, and ſhews the numbers it is placed between, are to be added together.

— Is the mark of *Subtraction*, and ſhews the latter of the numbers it ſtands between, is to be taken from the former.

× Is the mark of *Multiplication*, and ſhews the numbers on each ſide are to be multiplied together.

÷ Is the mark of *Diviſion*, and placed between two numbers, ſhews the former is to be divided by the latter. Or if one or ſeveral numbers are placed, part above a line, and part below it, it ſhews the upper are to be divided by the lower, and properly ſtands for the quotient.

= Is the mark of Equality, and ſhews the number or numbers preceding it, are equal to that which follows it.

√ placed before any figures, ſhews the Square root of them is meant. And

√c before a number denotes its Cube-root.

:: Is the ſign of *Proportion*, and being placed betwixt the two middle terms of four proportionals, ſignifies, *ſo is*, and : is generally put between the firſt and ſecond, and alſo between the third and fourth.

∴ is the mark for therefore.

To make all plain, the Characters are thus uſed:

$$2 : 4 :: 6 : 12\ \sqrt{c}\,4096 = 16,\ \sqrt{16} = 4,\ 4 + 2 = 6,$$
$$6 - 2 = 4\ \ 4 \times 2 = 8,\ \therefore 8 \div 2 = 4.$$

And thus read:

As 2 is to 4, ſo is 6 to 12; the Cube root of 4096 is, or is equal to 16; the Square root of 16 is equal to 4; 4 added to, or more by, 2, is equal to 6; and 6 made leſs by 2 is equal to 4; again, 4 multiplied by 2 is equal to 8; therefore 8 divided by 2 is equal to 4.

CHAP.

CHAP. XII.

Of Arithmetical and Geometrical PROPORTION, *or* PROGRESSION.

PROGRESSION being a rule more curious than useful, and requiring more words for its full explication, than are consistent with this *Compendium*, I shall only give some (the most necessary) propositions in both its parts, arithmetical and geometrical; and refer the more inquisitive reader to Mr. *Oughtred*'s *Clavis Mathematicæ*, for his further satisfaction.

In all continued proportion, or comparison of numbers, that number which is compared to another is called the *antecedent*, and the number to which it is compared is called the *consequent*.

There are two ways of comparing numbers with one another: 1*st*, with respect only to their simple difference, *i. e.* how much the one, *viz.* the *antecedent*, is greater, or less than the other, *viz.* the *consequent*, found by subtraction: thus the difference betwixt 5 and 8 is 3; which difference is called the *Arithmetical Ratio*. 2*d*. Numbers are compared with one another, when we consider how often one number contains, or is contained in another; and this is found by division, the quotient being the *geometrical ratio*. Thus, 2 is the *geometrical ratio* betwixt 8 and 16.

ARITHME-

ARITHMETICAL PROGRESSION.

ARITHMETICAL PROGRESSION is the regular increase or decrease of any rank of numbers by the continual addition or subtraction of some equal number.

So 1, 4, 7, 10, 13, 16, are in *Arithmetical Progression* to each other, equally increasing by the continual addition of 3.

And so also are 84, 82, 80, 78, regularly decreasing by the continual subtraction of 2.

Some of the most considerable properties of numbers in *Arithmetical Progression* are as follow:

1. That in any rank of numbers in *Arithmetical Progression*, consisting of three, or any odd number of terms, the double of the middle term will be equal to the sum of the two extremes, or of any two mean, equally distant from the said middle term.

1, 2, 3, 4, (5), 6, 7, 8, 9.

Thus in the rank above 5, the middle term doubled is 10, = to 1+9, the two extremes, or 2+8, or 3+7, or 4+6, the several correspondent means.

2. That in any rank of numbers in *Arithmetical Progression* consisting of four, or any even number of terms, the sum of the two extremes will be equal to the sum of the two middle numbers, or of any two means equally distant from the said extremes.

2, 4, 6, 8, 10, 12, 14, 16.

Thus in the rank above 2+16, the two extremes make 18, = 8+10, the two middle numbers, or 6+12, or 4+14, the other correspondent means.

In PROGRESSION *five things are to be noted.*

VIZ.

I. The first Term.
II. The last Term.
III. The number of Terms.
IV. The equal Difference.
V. The sum of all the Terms.

Any three of which being given, the other two may be found; as Mr. *Oughtred* in his *Clavis Mathematicæ, Chap,* 19. *Prop.* the 6th, exemplifies in 20 propositions; some of the most useful of which are as follow:

PROPOSITION. I.

The last term, number of terms, and equal difference being given, to find the first term:

Or, the second, third, and fourth given, to find the first.

RULE.

Multiply the fourth by the third, made less by one, and subtract the product from the second, the remainder is the answer.

EXAMPLE.

A man takes out of his pocket, at 6 several times, 6 several numbers of shillings, every one exceeding the former by 6; the last was 42, what was the first? *Answer*, 12*s*.

$6 \times 5 = 30$, $42 - 30 = 12$ *Answer*.

PRO-

PROPOSITION II.

The firſt, third, and fourth given, to find the ſecond.

RULE.

From the product of the fourth, multiplied by the third, ſubtract the fourth; the remainder added to the firſt, gives the ſecond.

EXAMPLE.

What's the laſt number of an *Arithmetical Progreſſion* beginning at 4, and continuing, by the increaſe of 12 to 18 places? *Anſwer*, 208.

12×18=216, 216−12=204, 204+4=208 *Anſwer.*

PROPOSITION III.

The firſt, ſecond, and fourth given, to find the third.

RULE.

Subtract the firſt from the ſecond, and divide the remainder by the fourth; the quotient, with one added, gives the third.

EXAMPLE.

A traveller went 12 miles the firſt day, and increaſed every day's journey by 4 miles, till at laſt he went 64 miles in one day; how many days did he travel? *Anſwer*, 14 days.

64−12=52, 52÷4=13, 13+1=14 *Anſwer.*

PROPOSITION IV.

The firſt, ſecond, and third given, to find the fourth.

RULE.

RULE.

Subtract the first from the second, and divide the remainder by the third, made less by one, the quotient gives the fourth.

EXAMPLE.

The ages of seven children increase by *Arithmetical Progression*; the youngest is two years old, the eldest 32: what's the common difference of their ages? *Answer*, 5 years.

$32-2=30$, $30 \div 7-1=5$ *Answer*.

PROPOSITION V.

The first, second, and third given, to find the fifth.

RULE.

Half the sum of the first and second, multiplied by the third, produces the fifth.

EXAMPLE.

12 persons give their charity to a poor man in *Arithmetical Progression*; the first gave 2 pence, the last 2 shillings, or 24 pence: how much did the poor man get? *Answer*, 156 pence, or 13 shillings.

$2+24=26$, $26 \div 2=13$, $13 \times 12=156$ *Answer*.

GEOMETRICAL PROGRESSION.

GEOMETRICAL PROGRESSION is the increasing or decreasing of any rank of numbers by some equal *ratio*, that is, by the continual multiplication or division of some equal number.

Thus,

Thus 2, 4, 8, 16, 32, increase in *Geometrical Progression*, by a double *ratio*, or continual multiplication by 2; and 405, 135, 45, 15, 5, decrease after the same manner by a continual division by 3, or by a triple *ratio*.

Some of the most considerable properties of numbers in *Geometrical Proportion* are as follow:

1. That in any rank of numbers in *Geometrical Progression*, consisting of three, or any odd number of terms, the product of the middle term, multiplied by itself, will be equal to the product of the two extremes, or of any two means equally distant from the said middle term.

1, 2, 4, (8), 16, 32, 64.

Thus, in the rank above 8, the middle term squared *i. e.* multiplied by itself, is $64 = 1 \times 64$, the two extremes, or 2×32, or 4×16, the correspondent means.

2. That in any rank of numbers in *Geometrical Progression*, consisting of four, or any even number of terms, the product of the two extremes will be equal to the product of the two middle numbers, or of any two means equally distant from the said extremes.

2, 4, 8, 16, 32, 64.

Thus in the rank above 2×64, the two extremes produce $128 = 8 \times 16$, the two middle numbers, or to 4×32, the other correspondent means.

In Geometrical Progression *the same five things are to be noted as in* Arithmetical Progression:

VIZ.

I. The first Term.
II. The last Term.
III. The number of Terms.
IV. The *ratio*, or equal difference.
V. The sum of all the Terms.

Note,

Note, Figures are sometimes set over a rank of geometrical proportionals, as here-under, and are called *Indices*, or *Exponents*; serving to shew the distance of any term from unity, or from the first term of the series.

Thus $\begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 1 . & 2 . & 4 . & 8 . & 16 . & 32 . & 64 \end{matrix}$

PROPOSITION I.

To find any remote term of a *Geometrical Progression*, proceeding from unity, whose *ratio*, or common difference is known, without producing all the intermediate terms.

RULE.

Find some few of the leading terms, and place over them their exponents; then multiplying the last found term by itself, it will produce a term double thereto; which again multiplied by itself, will produce another double to the last: Thus proceed till either you produce the term sought, or one a little short of it; which may be completed by multiplying it again by that term, which stands under such exponent as would make up the number.

Or, in other words, observe what figures of the exponents found added together, would give the exponent of the term wanted, and the numbers standing under the said exponents, multiplied into each other, will produce the term required.

From whence may be observed the coherence or agreement betwixt numbers in arithmetical and geometrical proportion.

EXAMPLE.

One agrees for 14 oranges, to pay only the price of the last, at a farthing for the first, an half penny the second, &c. still doubling the price for the next, what must he give?

Here, setting down only the five first terms, with their exponents, the fourteenth may thus be found, 5+5+4=14. So the term under those exponents being multiplied into each other, give the term answering. But as the exponent 1 stands always over the second term, so the exponent 13 answers to the 14th term; therefore add only 3 to the two 5's; and so always that exponent must be taken, which is one short of the term required; as for 20, 19; for 30, 29, &c. which must be carefully noted.

See the work of the foregoing example.

0	1	2	3	4	5
1	2	4	8	16	32

Exponents.	*Terms.*
5	32 *the 5th multiplied by*
5	32 *itself.*
3	64
13	96
	gives 1024 *the 10th, which*
	multiplied by 8 *the 3d Term,*
	gives 8192 *the 14th.*

And so many farthings, or 8 *l.* 10 *s.* 8 *d.* must he give at the rate agreed.

PROPOSITION II.

To find any remote term of a *Geometrical Progression*, not proceeding from unity, whose *ratio* and first term is given, without producing all the intermediate terms.

RULE.

RULE.

Set down, as before, some few of the leading terms with their exponents, and multiply also (as in the first proposition) the terms under such exponents, as added, would make the exponent short by one of the terms required; only remember, that every product must here be divided by the first term.

EXAMPLE.

A sum of money is thus to be divided among 9 persons, the 1st to have 50*l.* the 2d 150*l.* and so on in that proportion, one three times more than the other, to the last; what will the last have?

Setting down the five first terms with their exponents, thus,

0	1	2	3	4
50	150	450	1350	4050

Multiply the last term by itself, and divide the product by 50, the first term, the quotient will be 328050 the 9th term required, answering to the 8th exponent.

Or setting down but the three first terms, multiply the last by itself, and divide the product by the 1st term, it quotes the 5th; which again multiplied by itself, and divided by the first, will quote the 9th, as before. See the work.

0	1	2
50	150	450

$$\begin{array}{r} 450 \\ 450 \\ \hline 22500 \\ 1800 \\ \hline 5|0)20250|0(\\ \hline 4050 \end{array} \qquad \begin{array}{r} 4050 \\ 4050 \\ \hline 202500 \\ 16200 \\ \hline 5|0)1640250|0(\\ \hline 328050 \end{array}$$

Answer, 328050*l.* for the 9th person.

PROPOSITION III.

Having the firſt term, the common exceſs, and the number of places given, to find the total ſum of the whole progreſſion.

RULE.

Having found the laſt term by the foregoing rules, ſubtract from it the firſt, and divide the remainder by the common exceſs, made leſs by one; the quotient is the ſum of all, except the laſt: to which that being alſo added, gives the total required.

Note, If the *ratio* be double, that is, if the common exceſs be 2, the difference between the leaſt and the greateſt term, added to the greateſt, gives the total ſum.

If the *ratio* be triple, $\frac{1}{2}$ the difference muſt be added; if quadruple, $\frac{1}{3}$, *&c.*

EXAMPLES.

1*ſt*. A coffee-man, upon the ſigning of the laſt peace, for 50 guineas down, agreed to pay the firſt day one coffee-berry, the ſecond two, the third four, and ſo on, to double the quantity every day, till the ſame was proclaimed; what number of berries would it amount to, ſuppoſing the time 60 days: and what would their value be, ſuppoſing 100 berries to the pound, and the pound to be ſold at 5 *s* ?

See the work.

Days.

1—— 1 *Days.* 16 *Berries.*
2—— 2 16
3—— 4 96
4—— 8 16
5——16 256

2
512 *the* 10*th Day.*
16
3072
512
8192
2
16384 *the* 15*th Day.*
16348
65536
131072
49152
98304
16384
268435456
2
536870912 *the* 30*th Day.*
536870912
1073741824
536870912
4831838208
37580963840
4294967296
3221225472
1610612736
2684354560
288230370151711744
2
576460752303423488 *the* 60*th Day.*
1|000)1152921504606846|975 *Tot. number of berries.*
5 *s.* ¼ 288230376151711*l.* 10*s. their val.* at 5*s. per* ℔.

Ex. 2. A Grazier offers 40 oxen for a farthing a head, and treble it throughout; to what sum will it amount?

See the work.

Oxen.	*Farthings.*
1	1
2	3
3	9
4	27
5	81

```
   81
   81
 ----
   81
  648
 ----
 6561
    3
 ----
19683 10th Ox.
19683
-----
 59049
157464
118098
177147
19683
---------
387420489
        3
----------
1162261467 the 20th Ox.
```

```
                  1162261467
                  1162261467
                  ----------
                  8135830269
                 6973568802
                4649045868
               1162261467
              6973568802
             2324522934
            2324522934
           6973568802
          1162261467
         1162261467
         -------------------
         1350851717672992089
                           3
         -------------------
         4052555153018976267 40th Ox.
         -------------------
         6078832729528,64400 Tot. Far.
              Or,
         6332117426592150l. 8s. 4d.
```

CHAP.

CHAP. VIII.

Of VULGAR FRACTIONS.

A FRACTION is a part or parts of an unit, and written with two figures, with a line drawn between them, as $\frac{3}{4}$, $\frac{2}{3}$, $\frac{6}{8}$, each of which really stands for the quotient of a division made of the upper figure divided by the lower; so that if the upper figure can be multiplied into any number which the unit is equal to in known parts, and that product divided by the lower figure, the quotient will give the number of those parts equal to the fraction.

$$\text{As } 1l. = 20s.\ \tfrac{3}{4}l. = \tfrac{60}{4}s. = 15s.$$

Or,

$$1s. = 12d.\ \tfrac{3}{4}s. = \tfrac{36}{4}d. = 9d.$$

The figure under the line is called the *denominator*, because it gives name to the fraction, and also shews into what parts the unit is broken; and that above, the *numerator*, because it tells how many of such parts are meant by the fraction. Thus the fraction $\frac{3}{4}$ shews that the unit is divided into 4 parts, and that 3 of those parts are thereby express'd.

1. Therefore, if the *numerator* and *denominator* of a fraction should be equal, as $\frac{4}{4}$, $\frac{6}{6}$, the value of such fraction would be exactly an unit or integer; for, by the definition above, the *denominator* shews into how many parts the unit is broke: and the *numerator* expresses how many of those parts, are meant by the fraction: So that in the fraction $\frac{4}{4}$, the *denominator* declaring the unit to be broke into 4 parts, and the *numerator* expressing 4, that is, all of those parts, it is plain the said fraction is equal to an unit or whole number; because the sum of all the parts must be equal to the whole. From which observation it is also plain, that so often as the *denominator* is contain'd in the *numerator*, so many whole numbers or units are

contain'd in such a fraction. And this may serve as a reason for the operation of the 1st, 2d and 3d sorts of Reduction.

2. As all proper fractions may be supposed to arise from the remainders of divisions, when the divisor can no longer measure the dividend; so every fraction may be look'd upon as the two given terms of a division; the *numerator* as the dividend, and the *denominator* as the divisor, according to the latter part of the character, *page* 128, to signify division; from whence it appears that if the *numerator* and *denominator* of a fraction be either multiplied or divided both by the same number, the products or quotients will still remain in the same proportion, and the new fraction so arising be of the same value with that given. Thus the *numerator* and *denominator* of the fraction $\frac{2}{4}$, multiplied by 2, will produce $\frac{4}{8}$; or divided by 2, will quote $\frac{1}{2}$; all which fractions are of the same value; 4 bearing the same proportion to 8, and 1 to 2, as 2 does to 4. And from hence may appear the reason of the 5th and 6th sorts of Reduction.

Of Vulgar Fractions *there are four sorts*, viz.

1st. A *proper*; whose *numerator* is less than its *denominator*, as $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{8}$, *&c.*

2d, An *improper*; whose *numerator* is equal to, or greater than its *denominator*, as $\frac{5}{5}$, $\frac{6}{4}$, *&c.*

3d. A *compound*, or *fraction of a fraction*; known by the word *of*, as $\frac{1}{4}$ *of* $\frac{5}{8}$, *&c.*

4th. That which is join'd with a whole number, as $5\frac{1}{2}$, $3\frac{2}{3}$, *&c.* call'd a *mix'd number.*

Before fractions can be either added or subtracted, they must be reduced in o one denomination, that is, each to have the same *denominator*; and therefore, before we proceed to those rules, we must learn Reduction.

REDUCTION *of* FRACTIONS.

IN *Reduction of Fractions* there are ſeven varieties.

Firſt, To reduce a whole number into an *improper Fraction*; which is done by placing 1 for its *denominator*.

EXAMPLES.

Reduce 4 to a Fraction, *facit* $\frac{4}{1}$.
or 18,——————*facit* $\frac{18}{1}$.

But if you would aſſign it any other *denominator*, multiply the whole number by the *denominator* aſſign'd, and place the product for a *numerator* over the aſſign'd *denominator*.

EXAMPLES.

Reduce 12 to a fraction, whoſe *denominator* let be 8.
$\frac{8}{96}$ *facit* $\frac{96}{8}$.

Or if 6 were to be made a fraction, and its *denominator* to be 7, it would become $\frac{42}{7}$.

For the reaſon of this rule conſider the foregoing remarks.

Second, To reduce a mixed number into an improper one.

Multiply the integral parts of the number by the *denominator* of the fraction given, and take in the *numerator*; the product place for a new *numerator* over the *denominator* given.

EXAM-

EXAMPLES.

Reduce $5\frac{1}{3}$ to an improper fraction, *facit* $\frac{16}{3}$.

$$\begin{array}{r} 3 \\ \hline 16 \\ \hline 3 \end{array}$$

So $12\frac{1}{2}$ reduc'd to an improper fraction, will be $\frac{25}{2}$.

The reaſon of this rule is the ſame as the foregoing, there being no difference in the operation, but the taking in the given *numerator*.

Third, To reduce an improper fraction into its equivalent whole or mix'd number.

RULE.

Divide the *numerator* by the *denominator*, the quotient gives the whole number contain'd: But if any thing remains, (as in the ſecond example) it muſt be plac'd as a new *numerator* over the given *denominator*.

EXAMPLES.

Reduce $\frac{32}{8}$ to its equivalent whole number.

$$\begin{array}{l} 8)32(4 \text{ facit.} \\ \hline \quad 0 \end{array}$$

Reduce $\frac{45}{6}$ to its equivalent whole or mix'd number.

$$\begin{array}{l} 6)45(7\tfrac{3}{6} \text{ facit.} \\ \quad 42 \\ \hline \quad\; 3 \end{array}$$

This being only the reverſe of the foregoing rule, the ſame reaſon ſtill holds.

Fourth,

Fourth, To reduce a compound fraction to a ſingle one, *i. e.* a fraction of a fraction, to a fraction of an unit, which is the multiplication of them into one another.

RULE.

Multiply all the *numerators* together for a *numerator*, and all the *denominators* for a *denominator*, the fraction thus form'd is the product.

EXAMPLES.

Reduce $\frac{2}{3}$ of $\frac{1}{4}$ to a ſingle fraction, *facit* $\frac{2}{12}$.

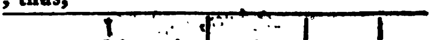
Reduce $\frac{4}{5}$ of $\frac{6}{8}$ of $\frac{7}{9}$ to a ſingle fraction, *facit* $\frac{168}{360}$.

See the work——

$$\begin{array}{cc} 4 & 5 \\ 6 & 8 \\ \hline 24 & 40 \\ 7 & 9 \\ \hline 168 & 360 \end{array}$$

So alſo $\frac{1}{8}$ of [illegible] is [illegible].

The reaſon of this operation will beſt appear by repreſenting the unit by a line, which, according to the firſt example, muſt be ſuppoſed to be divided into 4 parts, and each of thoſe parts again into 3 ſmaller parts; thus,



Then, as $\frac{1}{4}$ of the unit will denote 1 ſpace between the larger diviſions, ſo $\frac{2}{3}$ of that fourth muſt ſignify only 2 ſpaces of the leſſer diviſions; conſequently, if I would expreſs what part of the whole line $\frac{2}{3}$ of $\frac{1}{4}$ is, it will plainly appear to be $\frac{2}{12}$: or, had the compound fraction been $\frac{2}{3}$ of $\frac{3}{4}$, the ſingle fraction equal to it had been $\frac{6}{12} = \frac{1}{2}$. This ſhews that multiplying a fraction or any quantity by a proper fraction, decreaſes it in the ſame proportion as ſuch proper fraction is leſs than an unit.

Fifth, To reduce a fraction into its loweſt terms.

RULE.

Divide the *numerator* and *denominator* by any figure, ſo

so that nothing may remain, and the quotients will be a new fraction of the same value with that given.

The rule generally given for finding the greatest common measure, or number, to divide your given fraction by, is this:

Divide the *denominator* by the *numerator*, and the *numerator* by the *remainder*, if any: so continuing to make the last divisor the dividend, and the remainder the divisor, till nothing remains, the last divisor will be the greatest common measure, or number, by which you can divide your fraction.

EXAMPLE.

What is the greatest common measure by which $\frac{208}{684}$ can be divided?

```
          208)684(3
              624
              ---
               60)208(3
                  180
                  ---
                   28)60(2
                      56
                      --
Answer,                4)28(7
                         28
                         --
                          0
```

$\frac{208}{684}$ then, being divided by 4, gives $\frac{52}{171}$, which are its lowest terms.

But this way of finding the common measure is too tedious, often making more work than it saves. Observe therefore these more practical directions.

If you cannot at once discover the greatest number you may divide by, if both your *numerator* and *denominator* are even numbers, you may always halve them, and often that way reduce your fraction to advantage.

Thus

Thus the fraction given in the foregoing example, divided twice by 2, gives, as before, $\frac{52}{171}$.

$$2 \quad \frac{208}{684} \Big| \frac{104}{342} \Big| \frac{52}{171},$$

and $\frac{192}{576}$ may be reduced, by continual halving, to $\frac{3}{9}$.

$$2 \quad \frac{192}{576} \Big| \frac{96}{288} \Big| \frac{48}{144} \Big| \frac{24}{72} \Big| \frac{12}{36} \Big| \frac{6}{18} \Big| \frac{3}{9}.$$

Also, when both your *numerator* and *denominator* have cyphers on the right hand; you may abbreviate the fraction, by striking off an equal number from both.

Thus $\frac{45|00}{66|00}$, or $\frac{825}{990}$.

Or, if the right hand figures of your *numerator* and *denominator* are both fives; or one a five, and the other a cypher, as the remainder of the last example) you may always divide them by 5.

Thus $\frac{825}{960}$ divided by 5.
is $\frac{165}{192}$, equal to $\frac{825}{960}$.
So also is $\frac{125}{365}$ reduced to $\frac{25}{73}$.

Thus in the Rule of Three, we abbreviate our statings by the same rule, and on the same reason; for the second and third numbers multiply'd together, are as a *numerator* to the first number, their *denominator*.

For the reason of this rule, see the second remark.

Sixth, to reduce several fractions given with different denominators to have a common denominator.

RULE.

Multiply each of the given *numerators* into all the given *denominators*, except its own; and the several

products

products will be so many new *numerators*, whose common *denominator* must be the product of all the given *denominators* multiply'd together.

EXAMPLES.

Reduce $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{6}{10}$ to have a common denominator.

To do which,

First, The *numerator* 3 must be multiply'd by the *denominators* 8 and 10, which will produce 240.

Secondly, the *numerator* 5, multiply'd by the *denominators* 4 and 10, will produce 200.

Thirdly, The *numerator* 6, multiply'd by the *denominators* 4 and 8, will produce 192.

New numerators.

Then the three *denominators* 4, 8, 10, multiply'd together, will give 320 for a common *denominator*.

See the work.

1st Numb.	*2d Numb.*	*3d Numb.*	*Denomin.*
3	5	6	4
8	4	4	8
24	20	24	32
10	10	8	10
240	200	192	320

$\frac{240}{320}$ $\frac{200}{320}$ $\frac{192}{320}$

So $\frac{5}{10}$, $\frac{3}{8}$, $\frac{4}{12}$, and $\frac{9}{15}$ reduced to have a common Denominator will be $\frac{7200}{14400}$, $\frac{5400}{14400}$, $\frac{4800}{14400}$, $\frac{8640}{14400}$.

See the work.

1st Num.	*2d Num.*	*3d Num.*	*4th Num.*	*Deno.*
5	3	4	9	10
8	10	10	10	8
40	30	40	90	80
12	12	8	8	12
480	360	320	720	960
15	15	15	12	15
7200	5400	4800	8640	14420

The foregoing is the common Rule for reducing several fractions with different denominators, to have a common denominator, but as, by abbreviation fractions are often reduced into much lower terms, than they are at first given; so here, a common denominator may sometimes be found much smaller than that arising by the preceding method in the following manner.

RULE.

Having first, by the fifth sort of Reduction, reduc'd all your given fractions to their lowest terms, cancel or strike out all such of the *denominators* as are *aliquot* parts of others; then drawing a line under all the *denominators*, if two or more can be divided by any number, so that nothing may remain, divide by such number; and place the quotients under the line, and with them, in their proper places, all such of the *denominators* as would not admit of such division. If any of the said quotients or *denominators* so brought down, are capable of further abbreviation, draw another line, and proceed as before, so continuing as low as possible; then multiply continually, one into another, the several divisors, and the quotients arising, (with the remaining denominators if any) under the last line, and their product will be the least common *denominator*.

The

The new *numerators* to which may be found, by dividing the ſaid common *denominator* by each given *denominator*, and multiplying the quotients by their reſpective *numerators*.

EXAMPLE.

Reduce $\frac{3}{4}$ $\frac{5}{8}$ $\frac{6}{10}$ to have the leaſt common denominator 2——

$$\begin{array}{ccc} & 4 & 5 \\ \frac{30}{40} & \frac{25}{40} & \frac{24}{40} \end{array}$$

Anſwer, $\frac{30}{40}$ $\frac{25}{40}$ $\frac{24}{40}$.

The common *denominator* 40 is thus produced.

Firſt, The *denominator* 4 being an *aliquot* part of 8, is cancelled, then the *denominators* 8 and 10 are divided by 2. *Laſtly*, the diviſor $2\times4\times5=40$.

The reaſon of this ſixth ſort of reduction is evident from the 2d of the preceding remarks; for as both the *numerator* and *denominator* of each given fraction are equally multiply'd by all the other *denominators*, conſequently the new fractions thence ariſing, muſt be equal to the fractions given.

Seventh, To value a fraction, or reduce it to the known parts of an integer.

RULE.

Multiply the *numerator* by the next inferior denomination, and divide the product by the *denominator*; the quotient ſhews the parts ſought, and the remainder becomes a new *numerator* to the given *denominator*; which muſt ſtill be valued by the ſame rule, proceeding till you have brought it into the leaſt known parts of the integer.

EXAM-

EXAMPLES.

1*st*. What's the value of $\frac{3}{5}$ of a pound sterling?

```
   3
  20
  ——
5)60(
  12 Shillings.
```

2*d*. What's the value of $\frac{13}{49}$ of a pound sterling?

```
  13
  20
 ——— s. d. qrs.
49)260(5 . 3 . 2 34/49
 ———
  .15
   12
 ———
49)180(3
 ———
  .33
    4
 ———
49)132(2
 ———
  .34
```

3*d*. What hundreds, quarters, and pounds, are contained in $\frac{53}{94}$ of a ton?

```
   53
   20
 ——— C  qr. lb.
94)1060(11 . 1 . 2 92/94
   ———
   .120
   ———
    .26
      4
   ———
94)104(1
   ———
    .10
     28
   ———
94)280(2
   ———
    .92
```

There will be no difficulty in accounting for this rule, if we consider but the particular working of any one example: Thus in the first, $\frac{3}{5}$ of a pound sterling are given to be valued: Now as 20*s.* make a pound, so consequently any part of a pound must be 20 times as great a part of a shilling: Therefore $\frac{3}{5}$ of a pound make $\frac{60}{5}$ of a shilling; which being an improper fraction

tion its *numerator* is divided by its *denominator*, to find the units or whole numbers (which in this case must be shillings) contained in it; according to the directions of the 3d sort of Reduction.

By this rule are remainders of statings in the Rule of Three, &c. valued. See page 25.

ADDITION *of* FRACTIONS.

WHEN the given fractions are parts of one common denominator, all you have to do is, to add the numerators together, and place the sum for a new numerator over the given common denominator; thus $\frac{1}{6}$, $\frac{4}{6}$ and $\frac{3}{6}$ added together, make $\frac{8}{6}$ equal to $1\frac{2}{6}$.

But when amongst the given fractions there are either compound ones or single with different denominators, they must be prepared by reduction, before they can be added.

EXAMPLES.

1*st*. What's the sum of $\frac{3}{8}$, $\frac{2}{9}$, and $\frac{4}{7}$?

According to the sixth sort of reduction the given fractions being reduced to a common denominator they will be $\frac{189}{504}$, $\frac{112}{504}$, $\frac{288}{504}$, the numerators of which being added, the sum is $\frac{589}{504}$ equal to $1\frac{85}{504}$.

2*d*. Add $\frac{3}{4}$ of $\frac{3}{8}$, and $\frac{9}{10}$ of $\frac{6}{7}$ and $\frac{5}{8}$ together.

Reduce the compound fractions to single ones, by the fourth rule of reduction; and instead of $\frac{3}{4}$ of $\frac{3}{8}$, and $\frac{9}{10}$ of $\frac{6}{7}$, you will have $\frac{9}{32}$ and $\frac{54}{70}$, to add to your $\frac{5}{8}$; all which single fractions being reduced, first into their lowest terms, and then to a common denominator, they will be $\frac{315}{1120}$, $\frac{864}{1120}$ and $\frac{700}{1120}$; which added, make $\frac{1879}{1120}$, equal to $1\frac{759}{1120}$.

3*d*.

3d. Add $4\frac{1}{2}$ and $3\frac{1}{6}$ and $\frac{2}{7}$ together.

When mix'd numbers are given to be added, reduce only their fractional parts to one denominator, and add their sum to the total of the whole numbers.

$$
\begin{array}{llll}
\frac{42}{84} & \frac{14}{84} & \frac{24}{84} & 4 \text{———} 42 \\
 & & & 3 \text{———} 14 \\
 & & & \qquad\quad 24 \\
 & & & \qquad\quad \overline{} \\
 & & & \qquad\quad 80 \\
 & & \textit{facit}\ 7 & \text{———} \\
 & & & \qquad\quad 84
\end{array}
$$

More EXAMPLES.

Add 8 *l.* $\frac{1}{2}$ and 3 *l.* $\frac{9}{10}$ and $\frac{17}{15}$ together.

Facit 13 *l.* $\frac{4}{30}$.

What's the sum of $\frac{3}{4}$ of $\frac{7}{8}$, and 4 *l.* $\frac{1}{2}$ and $\frac{10}{30}$?

Answer, 5 *l.* $\frac{379}{480}$.

SUBTRACTION *of* FRACTIONS.

SUbtraction of Fractions is also nothing (after the given fractions are prepared by reduction) but taking on *numerator* from the other.

EXAMPLES.

1st. *From* $\frac{3}{4}$ *take* $\frac{4}{9}$.

$$
\begin{array}{r}
27 \\
16 \\
\hline
\end{array}
$$

prepared $\frac{27}{36}$ $\frac{16}{36}$ $\frac{11}{36}$ *Remainder.*

2d.

2d. *From* 5 $\frac{2}{3}$ *take* $\frac{9}{10}$.

First, The mix'd number 5 $\frac{2}{3}$ being reduced to the improper fraction $\frac{17}{3}$, by the second rule of reduction proceed as before.

From $\frac{17}{3}$, *take* $\frac{9}{10}$, or *prepared*.
From $\frac{170}{30}$ *take* $\frac{27}{30}$ *Remains* $\frac{143}{30}$ *or* 4 $\frac{23}{30}$.

Or without reducing the mixed nnmber to an improper fraction, make only the fractional parts of one denomination, and you will then have $\frac{27}{30}$ to take from $\frac{20}{30}$; which because you cannot do, you must borrow an unit, *i. e.* $\frac{30}{30}$; then, $\frac{27}{30}$ from $\frac{30}{30}$, there will remain $\frac{3}{30}$; to which adding the $\frac{20}{30}$, the remainder will then be $\frac{23}{30}$; and the unit borrowed being deducted from the whole number 5, there will remain in all 4 $\frac{23}{30}$, as before.

More EXAMPLES.

From 5*l.* $\frac{7}{8}$ *take* $\frac{9}{10}$ of $\frac{1}{6}$ *Rem.* 5*l* $\frac{1}{8}$
From $\frac{19}{20}$ *take* $\frac{1}{7}$ of $\frac{2}{3}$ *Rem.* $\frac{359}{420}$.

MULTIPLICATION *of* FRACTIONS.

IF the numbers given are whole, or mixed, they must first be brought into improper fractions; or if they are compound fractions, they must be reduced to single ones, or taken as part of the fractions to be multiplied together; but if the numbers given are either proper or improper single fractions, you have only to multiply the two given *numeraters* together for a new *numerator* and the given *denominators* for a *denominator*. The reason of which is clear, from the explanation of the fourth sort of reduction, which is this rule itself.

EXAM-

EXAMPLES.

1st. Multiply $\frac{4}{8}$ by $\frac{3}{6}$, *facit* $\frac{12}{48}$, or by abbreviation $\frac{1}{4}$.

2d. Multiply $\frac{9}{6}$ by $\frac{4}{7}$, *facit* $\frac{36}{42}$, or $\frac{6}{7}$.

3d. Multiply 4 by $\frac{3}{8}$. 4 by the first sort of reduction, reduced to a fraction, is $\frac{4}{1}$; which multiplied by $\frac{3}{8}$, makes $\frac{12}{8}$, or $1\frac{1}{2}$.

4th. Multiply $7\frac{1}{2}$ by $8\frac{2}{3}$. These mixed numbers reduced by the 2d rule of reduction to improper fractions, make $\frac{15}{2}$ and $\frac{26}{3}$; which, multiplied together, make $\frac{390}{6}$, or 65.

It may here be required to shew how mix'd numbers may be multiplied together without reducing them to improper fractions, which in many cases may shorten the work.

Suppose it, as in the last example, required to multiply $7\frac{1}{2}$ by $8\frac{2}{3}$.

First, 8 times $\frac{1}{2}$ or ($\frac{1}{2}$ of 8 is 4, without any fraction remaining, which carried to (8 times 7) 56 makes 60 to be set down as in multiplication of integers; now to multiply by $\frac{2}{3}$, we may first take $\frac{1}{3}$ of $7\frac{1}{2}$, thus, $\frac{1}{3}$ of 7 is 2, and 1 remains, which makes $\frac{3}{2}$ up $\frac{1}{2}$, and $\frac{1}{3}$ of $\frac{3}{2}$ is $\frac{1}{2}$, so $2\frac{1}{2}$ twice set down, and added to the 60, make up the product 65, as before.

$$\begin{array}{r} 7\frac{1}{2} \\ 8\frac{2}{3} \\ \hline 60 \\ 2\frac{1}{2} \\ 2\frac{1}{2} \\ \hline 65 \end{array}$$

Again,

Again, suppose it required to multiply $3\frac{3}{4}$ by $3\frac{3}{4}$.

First, 3 times $\frac{3}{4}$ is $(\frac{9}{4})$ $2\frac{1}{4}$. I set down the $\frac{1}{4}$, and carry the 2 to (3 times 3) 9, which makes 11 to be set down as an integer; now $\frac{3}{4}$ of $3\frac{3}{4}$ may be taken as in the rule of Practice, first the half of it, and then the half of that half, as the $\frac{1}{2}$ of 3 is 1; and one remains, that makes the $\frac{3}{4}$ up $\frac{7}{4}$, the half of $\frac{7}{4}$ is $\frac{7}{8}$; now half of $1\frac{7}{8}$ is $\frac{15}{16}$. These fractions are easily reduced, and added up with the integers.

$3\frac{3}{4}$
$3\frac{3}{4}$
———
$11\frac{1}{4}$
$1\frac{7}{8}$
$\frac{15}{16}$
———
$14\frac{1}{16}$

Let it be required to multiply $892\frac{3}{11}$ by $78\frac{7}{8}$.

First, 8 times 892 is 7136, and 70 times 892 is 62440; then taking the $\frac{7}{8}$ of 892 by several parts as in Practice, thus, half 892 is 446, half of 446 is 223, and half of 223 is $111\frac{1}{2}$; now for the $\frac{3}{11}$ of $78\frac{7}{8}$, first 3 times $78\frac{7}{8}$ is $236\frac{5}{8}$, which divided by 11, quotes $21\frac{45}{88}$, and these added is the product, as shewn at the side.

$892\frac{3}{11}$
$78\frac{7}{8}$
———
7136
6244
446
223
$111\frac{1}{2}$
$21\frac{45}{88}$
———
$70378\frac{1}{88}$

More EXAMPLES.

Multiply $9\frac{1}{2}$ by $\frac{2}{6}$, *facit* $3\frac{1}{6}$.
Multiply $\frac{7}{8}$ by 8, the product is 7.

This last example helps to clear up the reason given for the rule in Division, by shewing that multiplying a fraction by its denominator takes it away, and makes the numerator a whole number.

Division *of* Fractions.

WHEN you have made the ſame preparation of your number as directed in Multiplication, multiply the numerator of your dividend by the denominator of your diviſor, for a numerator; and the numerator of your diviſor by the denominator of your dividend, for a denominator; the fraction thus formed, is the quotient.

EXAMPLES.

1ſt. Divide $\frac{7}{9}$ by $\frac{3}{8}$, *facit* $\frac{56}{27}$, or $2\,\frac{2}{27}$.

2d. Divide $\frac{7}{8}$ by $\frac{1}{3}$, *facit* $\frac{21}{8}$, or $2\,\frac{5}{8}$.

3d. Divide 7 by $\frac{3}{4}$, prepared by the firſt rule of Reduction, it will be $\frac{7}{1}$ by $\frac{3}{4}$; *facit* $\frac{28}{3}$, or $9\,\frac{1}{3}$.

4th. Divide $4\frac{1}{2}$ by $2\frac{3}{8}$, by the 2d rule, it is $\frac{9}{2}$ by $\frac{19}{8}$, which divided gives $\frac{72}{38}$, or $1\,\frac{34}{38}$.

5th. Divide $\frac{2}{3}$ of $\frac{1}{9}$ by $\frac{4}{8}$ of $\frac{7}{8}$, reduced by the 4th Rule $\frac{2}{27}$ by $\frac{28}{64}$, *facit* $\frac{128}{756}$.

6th. Divide $9\frac{1}{2}$ by $\frac{2}{5}$ of $\frac{3}{4}$, *facit* $\frac{380}{12}$, $31\,\frac{2}{3}$.

The reaſon for this rule is, that the numerator divided by the denominator is the expreſs quantity of every fraction; therefore when one fraction is to be divided by another, we take the numerators as the numbers given, which muſt be cleared from being divided by their denominators, that is, we muſt take away thoſe diviſors, according to the laſt example in Multiplication; which is done by multiplying each numerator by the others denominator, the products whereof will be the numbers or numerators, cleared for diviſion, and both thereby increaſed in the ſame proportion.

$\frac{1}{2})\frac{3}{4}(\frac{6}{4}=1\frac{1}{2}$ Suppose $\frac{3}{4}$ to be divided by $\frac{1}{2}$, as at the side, here 3 is to be divided by 1; by the fractions $\frac{3}{4}$ and $\frac{1}{2}$, are the quotients of 3 divided by 4, and of 1 divided by 2; therefore, to clear each from being quotients, and to increase them in a like proportion, we multiply 3 by 2, and 1 by 4; which is really multiplying both the given fractions by 8, and produces $\frac{6}{4}$ equal to $1\frac{1}{2}$.

This shews, that when a fraction, or any quantity, is divided by a proper fraction, the quotient must be greater than the dividend, in the same proportion as the divisor is less than an unit.

If a mixed number is given to be divided by a whole number, the integral part of the dividend may first be divided; and, if any thing remains, multiply it by the denominator of the fraction, and add the numerator of the product for a numerator of the fractional part of the quotient, and place the product of the divisor so multiplied for a denominator; but, if such numerator is greater than the denominator, the excess is to be the numerator, and 1 added to the integral part of the quotient.

The DIRECT RULE *of* THREE IN VULGAR FRACTIONS.

AFter the numbers are prepared by reduction, state and work the question by the rules given in whole numbers.

EXAM-

EXAMPLES.

1*st*. What will ½ a pound of snuff cost, if 22 ℔. ¼ of the same come to 7 *l*. ⅓?

℔. l. ℔.

If 22 ¼ *cost* 7 ⅓, *what will* ½?

Prepared by the 2d rule of reduction, it will stand thus:

If $\frac{89}{4}$ . . $\frac{22}{3}$ . . $\frac{1}{2}$.

then the second and third numbers multiplied together produce $\frac{22}{6}$; which divided by $\frac{89}{4}$, gives $\frac{88}{534}$; which valu'd by the 7th rule of reduction, is 3*s*. 3*d*. 2 *qu*. $\frac{108}{534}$.

2*d*. Bought 87 gallons and an ½ of brandy for 40 *l*. at the same rate, what would ½ a gallon cost?

Gall. l. Gall.

If 87½ *cost* 40, what will ½?

prepared $\frac{175}{2}$ —— $\frac{40}{1}$ —————— $\frac{1}{2}$?

l. s. d. qu.

then $\frac{40}{1}\times\frac{1}{2}=\frac{40}{2}\div\frac{175}{2}=\frac{80}{350}=\frac{8}{35}$, or 4 . 6 . 3$\frac{3}{7}$.

Another RULE.

When your question is stated, and your numbers prepared, multiply the *denominator* of the first number and the *numerators* of the second and third continually, and place the product for a *numerator*. Then multiply the *numerator* of the first number and the *denominator* of the second and third continually, and place the product for its *denominator*; the new fraction so found, is the answer of the question. Take the stating of the last question for an

EXAMPLE.

If $\frac{175}{2}$ .. $\frac{40}{1}$ .. $\frac{1}{2}$ *facit* $\frac{80}{350}$.

For 2, the *denominator* of the first number, multiply'd by 40 and 1, the *numerators* of the second and third numbers, is 80.

And 175, the *numerator* of the first, multiply'd by 1 and 2, the *denominators* of the second and third, is 350. These products, placed fractionwise, make $\frac{80}{350}$, as before.

3*d*. What will 82 *C.* of sugar come to, if $\frac{3}{4}$ of $\frac{1}{8}$ of a *C.* cost $\frac{9}{10}$ of a pound Sterling?

C. *l.* *C.*

If $\frac{3}{4}$ *of* $\frac{1}{8}$ *cost* $\frac{9}{10}$, *what will* 82?

$\frac{3}{32}$ .. $\frac{9}{10}$ .. $\frac{82}{1}$

Facit $\frac{23616}{90}$, or 262*l.* 8*s.*

More EXAMPLES.

What will $\frac{1}{4}$ of an ounce of snuff cost, if the lb. comes to 8*s.* $\frac{1}{3}$?

Answer, 1 *d.* 2 *qua.* $\frac{1}{4}$.

At 1*d.* $\frac{1}{2}$ *per* ounce, what will 5 *C.* $\frac{1}{2}$ come to?

Answer, 61*l.* 12*s.*

The

The INDIRECT RULE *of* THREE IN VULGAR FRACTIONS.

HERE alſo you have only to obſerve the directions given in whole numbers:

Or this RULE.

After the numbers are ſtated and prepared, multiply the *numerators* of the firſt and ſecond numbers and the *denominator* of the third continually for a *numerator*; and the *denominators* of the firſt and ſecond numbers, and the *numerator* of the third continually for a *denominator*; the fraction ſo found is the *Anſwer*. See both ways in the example.

EXAMPLE.

How many yards of ſtuff, $\frac{1}{2}$ yard wide, are equal to $35\frac{1}{4}$ yards of $\frac{3}{4}$ wide?

$$\textit{If } \frac{3}{4} \text{———} 36\frac{1}{4} \text{———} \frac{1}{2}$$

$$\textit{Prepared } \frac{3}{4} \ \ldots\ldots\ \frac{145}{4} \ \ldots\ \frac{1}{2}$$

$$\frac{3}{4} \times \frac{145}{4} = \frac{435}{16},\ \frac{435}{16} \div \frac{1}{2} = \frac{870}{16},\ \textit{or } 54\tfrac{3}{8}.$$

So alſo 3, 145, and 2, the *numerator* of the firſt and ſecond numbers, and the *denominator* of the third, multiplied together, is 870.

And 4, 4, and 1, the *denominators* of the firſt and ſecond, and *numerator* of the third, multiplied together, produce 16.

Theſe numbers, fractionally placed, make $\frac{870}{16}$, as before.

More *E X A M P L E S.*

If *A.* lends *B.* 35*l.* $\frac{1}{3}$ for 6 months $\frac{1}{2}$, how long may he keep 10*l.* $\frac{1}{4}$ of *B*'s, to requite himself?

Answer, 22 months, $\frac{50}{123}$.

If *A.* keeps 100*l.* $\frac{1}{2}$ of B's 4 months, $\frac{1}{3}$, what sum must *A.* lend *B.* for 2 years $\frac{1}{2}$ to requite him?

Answer, 14*l.* $\frac{253}{540}$.

The COMPOUND RULE *of* THREE IN VULGAR FRACTIONS.

TO the Rules before given page 42, let it be observed, that where any of the terms given is a fraction (or a mixed number, which must be put down as an improper fraction) in the operation, the denominator will be a factor contrary to the numerator; and when an equation is made for the quotient Q, where the several factors of the dividend are put as a numerator, and the several factors of the divisor as a denominator; then whatever factors in each are alike, they may be struck out, and for those equally divisible take their quotients, whereby the operation will frequently be very much shortned.

EXAM-

EXAMPLE.

If 264 men, in $5\frac{1}{4}$ days working $10\frac{1}{2}$ hours each, dig a trench $234\frac{2}{3}$ yards long, $3\frac{1}{4}$ wide, and $2\frac{7}{8}$ deep; in how many days working, $9\frac{3}{4}$ hours each, will 25 men dig a trench $337\frac{1}{2}$ yards long, $4\frac{1}{3}$ wide, and $3\frac{3}{4}$ deep?

men	*days.*	*hours.*	*long.*	*wide.*	*deep.*
If 264	$\frac{21}{4}$	$\frac{21}{2}$	$\frac{704}{3}$	$\frac{13}{4}$	$\frac{23}{8}$
25	Q	$\frac{39}{4}$	$\frac{675}{2}$	$\frac{13}{3}$	$\frac{15}{4}$

Here as in *Ex.* 5. page 44, the trenches are the produced terms, and the men, days and hours the producing; therefore

$$Q = \frac{675 \times 13 \times 15 \times 264 \times 23 \times 21 \times 4 \times 3 \times 4 \times 8}{704 \times 15 \times 23 \times 25 \times 39 \times 4 \times 2 \times 2 \times 3 \times 4} = \frac{27 \times 21}{4}$$

$$= \frac{567}{4} = 141\tfrac{3}{4} \textit{ days.}$$

THE ARITHMETIC OF DECIMAL FRACTIONS.

THIS part of Arithmetic is very compendious and easy in operation, and therefore very useful, especially in calculations of Interest, valuing annuities, &c. But, by reason some fractions cannot be exactly expressed by decimal parts, it may not always be convenient to work with them. Mr. *Cunn* hath, in his late treatise on fractions, very curiously shewn how they may be used with least loss; but, since such exactness destroys the brevity, I cannot but think, where great perfection is required, vulgar fractions preferable.

CHAP.

CHAP. I.

Of Numeration, Addition, Subtraction, Multiplication, and Division of Decimal Fractions.

Numeration.

IN Decimal Fractions, the figures expressed with a point or comma prefixed is the numerator, and 1 with as many cyphers as the numerator hath places is always the denominator, and therefore known without being set down; thus, ,5 stands for $\frac{5}{10}$, ,25 for $\frac{25}{100}$, ,123 for $\frac{123}{1000}$, &c. wherein we imagine the integer or unit (whether it be time, coin, weight, measure, &c. as a year, a pound sterling, a pound weight, a bushel, a yard, a foot, &c.) to be divided into 10, 100, 1000, &c. equal parts at pleasure; for dividing it first into 10 equal parts, and then each of those into 10 lesser equal parts, will divide the unit into 100 equal parts; and if again we divide each of those hundredths into 10 equal parts, the unit will be divided into 1000 equal parts, and so on infinitely.

The different value of the several places will more plainly appear from their explication under the following number.

Integral Parts.						Fraction Parts.					
6	5	4	3	2	1,	2	3	4	5	6	7.
C Thousands.	X Thousands.	Thousands.	Hundreds.	Tens.	Units.	Tenths.	Hundredths.	Thousands.	X Thousands.	C Thousands.	Millions.

From whence it is evident,

That as the significant figures or parts in the whole numbers increase by a ten-fold proportion towards the left-hand from the units place; so, in Decimal Fractions, they decrease in the same proportion towards the right; therefore cyphers at the left-hand in a decimal fraction decrease the value in a tenfold proportion, by removing those significant figures farther from the unit's place.

$$\text{Thus}\left\{\begin{array}{l} ,5 \text{ is five tenth parts} = \tfrac{5}{10} = \tfrac{1}{2} \\ ,05 \text{ is five hundredths} = \tfrac{5}{100} = \tfrac{1}{20} \\ ,005 \text{ is five thousandths} = \tfrac{5}{1000} = \tfrac{1}{200} \end{array}\right\} \text{of an unit.}$$

But cyphers at the right-hand of a decimal fraction alter not its value, because the places of the significant figures thereof are not thereby changed; for ,5 ,50 ,500 &c. are each $\frac{5}{10}$ of an unit, the 5 in each being in the first place of fractions, *viz.* the place of tenths.

These things understood, the rest is easy.

ADDITION.

HAVING set down your proposed numbers, units under units, and fraction places under places of like value, add them as if they were all whole numbers, separating so many fractional places in the total, as were in that given number which had the most.

EXAM-

EXAMPLES.

Add 5,42, 38,583, 421,06, and ,8345 together.

These numbers,	5	,42
rightly placed,	38	,583
will stand thus:	421	,06
		,8345
Total,	465	,8975

What's the sum of 4,06 *l.*, 21,734 *l.*, and 523,8642 *l.*

523,8642
21,734
4,06
Answer, 549,6582 *l.*

SUBTRACTION.

HAVING placed the numbers as directed in addition, work still as if they were all whole numbers.

EXAMPLES.

From 839,38275 *l.* *take* 10,49561 *l.*

839,38275
10,49561
828,88714 *Remainder.*

Take 23,279 *Yards,* *from* 451,15842 *Yards.*

451,15842
23,279
Remains, 427,87942

MULTI-

MULTIPLICATION.

HERE the numbers are to be set down, without regard to the value of their places, and worked in all respects the same as in whole numbers; only observe to separate so many places of fractional parts at the right-hand in the product, as were in both multiplicand and multiplier.

EXAMPLES.

```
Multiply     2,735641        Multiply      73,82564365
by            ,54382         by             8,7434592
          ----------                   ---------------
           |5471282                    |14765128730
         21|885128                     |66443079285
         82|06923                     3|6912821825
       1094|2564                     29|530257460
       1368|205                     221|47693095
  ---------------                  2953|0257460
    1,4876|9628862                51677|950555
                                 590605|14920
                               ---------------
                       Product  645,491|50316751 4080
```

A compendious way to multiply numbers that have many fractional places; by omitting all the figures to the right of the perpendicular lines in the examples above.

RULE.

Place one of the numbers just as it is given, for a multiplicand, and under that fractional place which you would have be the last in the product, set the unit's place of your other number; then, reversing all the other figures, multiply with them in their order, beginning always with that figure of the multiplicand, which stands over the number you work with; having also respect to the increase you think would have

been brought from the omitted figures on the right-hand, had they been multiplied.

Note, The usual allowance for such omitted figures is as follows, *viz.* If the next figure on the right-hand of that you begin with in the multiplicand, multiply'd into the figure of the multiplier you are working with, gives a product between 5 and 15, carry 1; if the product be above 15, and less than 25, carry 2; and if it arise to any number between 25 and 35, carry 3, &c.

To prove the certainty of the rule, we will take the same example before given.

EXAMPLES.

Multiply 2,735641 by ,54382, and produce only four places of decimals.

```
2,735641
28345,0
-------
13678 the Increase here carried is 3 for 5×6
 1094 the Increase here is 2 for 4×5
   82 the Increase here is 1 for 3×3
   22 the Increase here is 6 for 8×7
------
1,4876
```

Note, There being no units in the multiplier, its place is supplied with a cypher.

Multiply 73,82564365 by 8,7434592, and produce but 3 places of decimal fractions.

```
 73,82564365
2954347,8
---------
  590605
   51678
    2953
     222
      29
       4 the Increase carried for 5×7.
---------
 645,491
```

Note,

Note, To multiply any decimal fraction by 10, 100, 1000, &c is only to remove the mark of separation so many places towards the right-hand as there are cyphers.

Thus, 8,3564537 multiplied by	10,	is	83,564537
	100,		835,64537
	1000,		8356,4537
	10000,		83564,537, &c.

DIVISION.

THIS rule also being work'd the same as in whole numbers, the only difficulty is to value the quotient; which may be done by either of the following general rules, *viz.*

RULE 1.

The first figure in the quotient is always to stand in the same place, with that figure of the dividend, which answers, or stands over the place of units in the divisor, Or,

RULE 2.

The quotient must always have so many fractional places, as the dividend has more than the divisor.

Note, 1*st*. If the divisor and dividend have both the same number of fraction places (as in the second example) the quotient will be all a whole number.

2*d*. If the dividend hath not so many places of fractions as are in the divisor, as in example the 3d.) so many cyphers must be annexed to the dividend, as will make them equal; and the quotient will then, as before, all be a whole number.

3*d*. But if, when division is done, the quotient has not so many figures as it should have fraction places (as in the last example) so many cyphers must be prefix'd, as there are places wanting.

EXAM-

EXAMPLES.

1*st*. Divide 10,55439 by 4,569.

4,569)10,55439(2,31

14163

4569

...

Here, according to the first rule, 4, the unit's place of the divisor, standing under units, or the first place of whole numbers in the dividend; 2, or first figure plac'd in the quotient, must also be made the first place of whole numbers, by placing the point of separation immediately after it.

Or, according to the second rule, there being five places of fractions in the dividend, and but three in the divisor; two places must be separated off in the quotient, to make up the number.

Ex. 2*d*. Divide 85643,825 by 6,321.

6,321)85643,825(13549

22433

.34708

.31032

57485

.596

Here 6, the place of units in the divisor, standing under 8, the fifth place of the whole numbers in the divi-

dividend, 1, the first figure of the quotient, must also stand in the same place, which makes all the quotient whole numbers.

And so it must also be by the second rule, because the dividend has no more places of fractions than the divisor.

Ex. 3. Divide 7382,54 by 6,4352.

6,4352)7282,5400(1147

Here there not being so many places of fractions in the dividend as in the divisor, two cyphers are added to make them even; and being so, the quotient is all a whole number as before.

Ex. 4. Divide ,08516438 by 423.

```
423),08516438(,00020133
        564
       ----
       1413
       ----
       1448
       ----
        179
```

In this example, the quotient consisting but of five figures, whereas there should, by the rules, be eight fraction places of decimal parts, three cyphers are prefixed to make up the number, as was directed in *Note* the third.

Then

There is also a way of contracting the work of a large division, somewhat like that compendious method before taught, of multiplying numbers of many places of decimal fractions, and bringing out only so many in the product as are necessary for the answer required.

RULE.

Having consider'd, by the first rule given for valuing the quotient, in what place the first figure thereof ought to stand, and accordingly determined how many figures it is necessary it should consist of, you may from thence judge whether any, or how many of the right-hand figures of your divisor may be neglected; so that reserving only so many figures of the dividend as are necessary for once answering the divisor, leave out the rest to the right-hand as useless; and having set the proper figure in the first place of the quotient, work with it as usual; then omitting the right-hand figure of your divisor, seek how often the other figures of your divisor are contained in the remainder; which figure being entered in the quotient, and work'd with as usual in division, (with regard, however, to the carriage that would have been brought from the omitted figures, as before directed in multiplication) thus continuing to neglect the right-hand figure (putting a point under for ready knowing it) of your divisor every time you seek a new quotient figure, you will still be able to divide the remainder left after the last subtraction by the divisor so lessened, till you have brought out the determined number of figures in the quotient.

The following example, which is set down both at length, and the contracted way, will make all clear.

EXAM-

EXAMPLE.

```
3,4637528)95,4327|56463275(27,55188
          69275C|56
          ------|---
          261577|004
          242462|696
          ------|---
           19114|3086
           17318|7640
          ------|---
            1795|54463
            1731|87640
          ------|-----
              63|668232
              34|637528
          ------|------
              29|0307047
              27|7100224
          ------|-------
               1|32068235
          ------|--------
```

```
3,46375)95,4327(27,5518
        69,2750
        -------
        26,1577
        24,2463
        -------
          19114
          17319
          -----
           1795
           1732
           ----
             63
             35
             --
             28
             27
             --
              1
```

And thus are all the figures on the right-hand of the perpendicular line in the example work'd at length, sav'd by working after this contracted manner.

As multiplying by 10, 100, 1000, &c. is only removing the separating point of the multiplicand, so many places towards the right-hand as there are cyphers in the multiplier; so, to divide by 10, 100, 1000, &c. is only removing the mark of separation, after the same manner, towards the left-hand.

Thus 83564,537 divided by	10,	is	8356,4537
	100,		835,64537
	1000,		83,564537
	10000,		8,3564537

These examples being, I hope, sufficient to make division plain, we will now proceed to *Reduction of Decimals.*

CHAP.

CHAP. III.

Of Reduction of Decimals.

THERE are two sorts of *Reduction of Decimal Fractions*. 1*st*. Changing a vulgar fraction to a decimal. 2. Valuing a decimal fraction by the known parts of an integer.

1*st*. To reduce or change a vulgar fraction to a decimal.

RULE.

Place cyphers at pleasure after the *numerator*, and divide by the *denominator*.

EXAMPLES.

Reduce $\frac{1}{4}$ to a decimal, 4)1,00(,25 *facit*.

What decimal parts are equal to $\frac{3}{8}$?

8)3,00(,375 *facit*.

And so any part of coin, weight, measure, time, &c. may be brought into a decimal, by first reducing it into a vulgar fraction which is done by placing the known parts of the integer as a denominator under the parts given) then annexing cyphers to the *numerator*, and dividing by the *denominator*, as before directed.

EXAM-

EXAMPLES.

What decimal parts of a pound are equal to 16*s.*?
i. e. $\frac{16}{20}$ 20)16,0(,8 *facit.*
What decimal parts of a ℔. *Troy*, are equal to 9 *oz.*?
i. e. $\frac{9}{12}$ 12)9,00(,75 *facit.*

Note, If the given parts are of several denominations, they may be reduced either by so many distinct operations as there are different parts; or by first reducing them into their lowest denomination, and then one division will serve. See both ways in the

EXAMPLES.

Reduce 14 *s.* 9 *d.* ¼ into decimal parts of a pound Sterling: By the first way there will be these three vulgar fractions to be changed, $\frac{14}{20}$, $\frac{9}{240}$, and $\frac{3}{960}$.

20)14,00(,70 240),90000(,0375 ,7
,0375
960)3,0000(,0031 1800 ,0031
.1200 1200 *facit* ,7405
.240 .00

The other way. 14 *s.* 9¼ *d.* *being reduced to farthings.*
12
177
4
will be 711
which divided by its denominator } 960)711,0000(,7406 *is produced as* (*before.*
.3900
.6000
.240

What

What decimal parts of a C. *wt.* are equal to 3 *qrs.* 18 lb.?

By the first way.

$\frac{3}{4}$ and $\frac{18}{112}$ 4)3,00(,75 — 112)18,0000(,1607
,1607 .680
—— ——
Answer, ,9107 ..800
——
.16

By the second way.

qrs. lb.
3 18 *Answer*, 112)102,0000(,9107
28 ——
—— .120
102 ——
—— .800
112 *Denominator.* ——
.16

There is also another way of reducing parts of different denominations into decimals of the highest integer; thus,

RULE.

Bring the lowest parts into decimals of the next superior denomination; and on the left-hand of the decimal fraction found, place the parts given of that said next superior denomination; so proceeding, till you bring out the decimal parts of the highest integer requir'd, by still dividing the product by the next superior *denominator*. For examples, take those before given, *viz.*

Reduce

Reduce 14*s.* 9*d.* ¾ into decimal parts of a pound Sterling.

4	3,
12	9,75
20	14,8125
	,7406

Here the 3 farthings being divided by 4, gives 75, to which the 9*d.* being prefixed, it becomes 9,75; which divided again by 12, quotes ,8125; before which the 14*s.* being also placed, it makes 14,8125; which being lastly divided by 20, the quotient ,7406, is the answer.

What decimal parts of a *C. wt.* are equal to 3 *qrs.* 18 lb?

28 {	7	18,
	4	2,5714
	4	3,6428
		,9107 *Answer.*

Note, Instead of dividing by 28, which would be troublesome; 'tis better to divide by 7 and 4, as in the example, 4 times 7 being 28.

Thus you see how easily the decimals, answering to any given parts of coin, weight, measure, &c. are found; and after the same manner are the following tables formed.

But shillings, pence, and farthings, may more readily be reduc'd thus:

RULE.

For the shillings (if their number be even) set down their half in the first place of decimal fractions, and let the second and third places be filled up with the farthings contain'd in the remaining pence and farthings, always remembring to add one, when the number

ber is or exceeds 25; but if the number of shillings be odd, the second place must also be increased by 5.

Thus 8*s*. 5*d*. ½, reduced to a decimal fraction, is ,422; 4 in the first place standing for 8*s*. and 22 farthings being 5*d*. ½.

Again, 8*s*. 6*d*. ½, will be ,427.

1 being added to 26, the number of farthings in 6*d*. ½, because exceeding 25.

And lastly, 9*s*. 6*d*. ½, express'd decimally, is ,477.

5 being added to the second place, because the number of shillings is odd, 1 shilling being ,05.

The second sort of reduction is, to value a decimal fraction by the known parts of its integer; which is thus done:

RULE.

Multiply the decimal fraction given by the known parts of the next inferior denomination, and separating the product as directed in multiplication, then multiply the decimal fraction of the product by the known parts of the next inferior denomination; thus proceeding till you have brought it into the least known parts of the integer, the figures standing on the left of the separating points will be the parts required.

EXAMPLES.

Ex. 1. What's the value of ,7691, the integer a pound sterling? (*i. e.*) What shillings, pence, and farthings are equal to ,7691 parts of a pound sterling?

	,7691	*parts of a pound*
multiplied by	20	*the shillings in* 1*l*.
produce shillings	15,3820	*and parts of a shill.*
which multiplied by	12	*the pence in* 1 *shill.*
make pence	4,5840	*and parts of a penny.*
which again multiplied by	4	*the farthings in* 1*d*.
give farthings	,3364	*and parts of a farth.*

So

So that the value of ,7691 decimal parts of a pound sterling is 15*s.* 4*d.* $\frac{1}{2}$.

Ex. 2. What ounces, penny weights, and grains, are equal to ,84362 decimal parts of a pound *Troy?*

	,84362	*parts of a pound* Troy,
multiplied by	12	*the ounces in one pound.*
give ounces	10,12344	*and parts of one ounce.*
multiplied by	20	*the penny weights in* 1 *oz.*
give penny-weights	2,46880	*and parts of* 1 *penny-wt.*
multiplied by	24	*the grains in* 1 *penny-wt.*
	187520	
	93760	
Give grains	11,25120	*parts of* 1 *grain.*

The Answer *therefore is* 10 *oz.* 2 *dwts.* 11 *grs.*

But the decimal parts of a pound sterling may be thus valued at sight:

RULE.

The figure standing in the first place of decimal parts doubled, gives shillings; but if the figure in the second place is or exceeds 5, one more must be added to their number; the second figure, (if under 5) or its excess, (if above 5) joined with the third, are so many farthings; only remember to abate 1, if their number amounts to near 25; or 2, if near 50.

The reason of this abatement is, that as 1000 is the *denominator* of every decimal fraction consisting of 3 places, so consequently by reckoning the figure standing in the said third place, as so many farthings, we thereby allow 1000 farthings to the pound; whereas indeed there are

are but 960: The overplus therefore being 40 in 1000, is 4 in a 100, or 1 in 25, &c. according to the direction above.

EXAMPLES.

Ex. 1. What's the value of ,375 parts of a pound sterling?

Answer, 7*s*. 6*d*.

The 3 doubled is 6 shillings; to which 1 *s*. being added for the 5, in the second place, makes it 7*s*. and 2 remaining, join'd with the 5 in the 3d place, being accounted 25 farthings, from which 1 being deducted, there remain 24 farthings, or 6 pence.

Ex. 2. What's the value of ,846 parts of a pound?

Answer, 16*s*. 11*d*.

The 8 doubled, making 16*s*. and 2 abated from the 46 farthings, leaves 44 farthings, or 11*d*.

Thus are shewn the ways both of finding the decimal fractions answerable to any given parts of coin, weight, measure, &c. or the value of any given decimal in the known parts of its integer.

I have likewise annexed TABLES ready calculated by the foregoing rules, answering to the same ends.

The

The uſe of which is very plain; one column in each table containing the number of ſhillings, ounces, drachms, pints, buſhels, days, hours, or any thing elſe you may want; and againſt the ſaid number, in the other column, the decimal parts anſwering: Thus againſt 3 pence in the firſt table, is found ,0125, and againſt 40 days in the fourth table ,109589.

But if the number you want is not to be found in the ſaid tables (many compound numbers having been omitted, to reduce them into ſo narrow a compaſs) you muſt add the parts of ſuch two or three numbers together, as will compoſe it, and the total will be the decimal fraction ſought. Thus, if the decimal fraction anſwering to 17 penny-weights, the integer a pound *Troy*, were to be ſought in the 2d table, ,04166, the decimal fraction anſwering to 10 penny-weights, and ,029166, that anſwering to 7 penny-weights, and added together, the total ,070832, will be found the anſwer; or, if in table the fourth, you would find the decimal of 352 days,

$$\begin{array}{rl} \textit{add}\ ,821918 & \left.\begin{array}{l}\\ \\ \\ \end{array}\right\} \textit{the decimal fractions of} \left\{\begin{array}{r} 300 \\ 50 \\ 2 \end{array}\right\} \textit{Days.} \\ ,36986 & \\ ,005479 & \\ \hline \textit{total}\ ,964383 & \qquad \textit{anſwering 352 Days.} \end{array}$$

And ſo of any other.

CHAP.

Decimal Tables of Coin & Weight

Table 1st — Coin, 1 £ ster: yᵉ Integer

s	Dec:	s	Dec:
19	.95	9	.45
18	.9	8	.4
17	.85	7	.35
16	.8	6	.3
15	.75	5	.25
14	.7	4	.2
13	.65	3	.15
12	.6	2	.1
11	.55	1	.05
10	.5		

Pence	Decim:
11	.045833
10	.041666
9	.0375
8	.033333
7	.029166
6	.025
5	.020833
4	.016666
3	.0125
2	.008333
1	.004166

Farthings	Decim:
3	.003125
2	.002083
1	.001042

Table 2d — Troy Weight, 1 lb the Integer

Ounces	Decim:
11	.916666
10	.833333
9	.75
8	.666666
7	.583333
6	.5
5	.416666
4	.333333
3	.25
2	.166666
1	.083333

Note This Table of Oz: will also serve for Inches Months or Doz:

Penny wt	Decim:
10	.041666
9	.0375
8	.033333
7	.029166
6	.025
5	.020833
4	.016666
3	.0125
2	.008333
1	.004166

Grains	Decim:
20	.003472
10	.001736
9	.001562
8	.001389
7	.001215
6	.001042
5	.000868
4	.000694
3	.000521
2	.000347
1	.000173
½	.000086

Table 3d — Avoirdupoise, 112 lb yᵉ Integer

Quarters	Decim:
3	.75
2	.5
1	.25

Pounds	Decim:
20	.178571
10	.080286
9	.080357
8	.071428
7	.0625
6	.053571
5	.044643
4	.035714
3	.026786
2	.017857
1	.008928

Ounces	Decim:
10	.005580
9	.005022
8	.004464
7	.003906
6	.003348
5	.002790
4	.002232
3	.001673
2	.001116
1	.000558

Drams	Decim:
10	.000348
9	.000313
8	.000279
7	.000244
6	.000209
5	.000174
4	.000139
3	.000104
2	.000069
1	.000034
½	.000017

Webster scripsit. Bickham Sculpsit.

Decimal Tables of Measure and Time

Table 4th. Time — 1 year ye Integer

Days	Decim:
300	.821918
200	.547945
100	.273973
90	.246575
80	.219178
70	.191781
60	.164383
50	.136986
40	.109589
30	.082192
20	.054794
10	.027397
9	.024657
8	.021918
7	.019178
6	.016438
5	.013699
4	.010959
3	.008219
2	.005479
1	.002739

Table 5th. Measure — Liquid. Dry. Integer 1 Gall: 1 Quarter

Pints	Decim:	Bush
7	.875	7
6	.75	6
5	.625	5
4	.5	4
3	.375	3
2	.25	2
1	.125	1

qr Pts	Decim:	Pecks
3	.09375	3
2	.0625	2
1	.03125	1

Decim:	qr Pecks
.023437	3
.015625	2
.007812	1

Decim:	Pints
.005859	3
.003906	2
.001953	1

Cloth Measure — 1 Yard ye Integer

Quarters	Decim:
3	.75
2	.5
1	.25

Nails	Decim:
3	.1875
2	.125
1	.0625

Table 6th. Time — 1 Day ye Integer

Hours	Decim:
20	.833333
10	.416666
9	.375
8	.333333
7	.291666
6	.25
5	.208333
4	.166666
3	.125
2	.083333
1	.041666

Minutes	Decim:
50	.034722
40	.027777
30	.020833
20	.013888
10	.006944
9	.00625
8	.005555
7	.004861
6	.004166
5	.003472
4	.002777
3	.002083
2	.001388
1	.000694

Webster Scrip.t — Bickham sculp.t

CHAP. III.

Of the RULE *of* THREE.

THIS *Rule of* PROPORTION having been already sufficiently explained, both as to its nature and operation, in *Vulgar Arithmetic*, 'tis needless to repeat here what has been there said. Only therefore observe, that instead of preparing your numbers, by reducing them into their lowest denomination, you must now bring their fractional parts into decimals. See the examples.

EXAMPLES.

Ex. 1. If the price of 26 yards and $\frac{1}{2}$ of drugget is 3*l.* 16*s.* 3*d.* what will 32 yards $\frac{1}{4}$ come to?

The fractional parts of the numbers being reduced to decimals, and the question stated as usual, the work will stand as follows;

Yards. *l.* *Yards.*
If 26,5—3,8125—32,25

```
             32,25
            ------
            190625
            76250
           76250
          114375
         ---------
26,5)122,953125(4,63974
         ----
          1695
          ----
           1053
           ----
            2581
            ----
             1962
             ----
              1075
              ----
               ..15
```

The quotient valued by the short rule for money, makes 4 l. 12 s. 9 d. ½.

Ex. 2. What will the pay of 540 men amount to, at 1*l.* 5*s.* 6*d.* *per* man?

Man. *l.* *Men.*
If 1———1,275———540

```
          540
        -----
        51000
        6375
        ------
```

Answer, 688,500 *or* 688*l.* 10*s.*

For more examples, take those given in *Vulgar Arithmetic.*

The

The Indirect RULE *of* THREE.

THE Indirect *Rule of Three* is also the same here as in *Vulgar Arithmetic*, differing only in the preparation of the numbers.

EXAMPLE.

What must the penny-loaf weigh, when wheat is 10*s*. the bushel, if, when at 6*s*. 5*d*. *per* bushel, the penny-loaf weighs 5 *oz*.½ ?

If ,32083 ——— 5,5 ——— ,5
5,5
160415
160415
,5)1,764565(*oz. dwts. grs.*

Answer, 3,52913 *oz*.=3 .. 10 .. 14 nearly.

Note, Where the last figure of a Decimal Fraction would recur infinitely, and it be required to have the operation exact, annex the Vulgar Fraction in that place, *viz*. that figure numerator with 9 denominator or their equal, and in any multiplication thereof, add such part of the multiplier in the lowest place to make up the product. Suppose in the example,

l. *l*. *oz*. *l*.
If ,3208⅓=,3208⅓ —— 5,5 —— ,5
5,5
16040
16040
18⅓
,5)1,76458⅓ *oz. dwts. grs.*

Answer, 3,5291⅔ *oz*.=3 .. 10 .. 14 more near.

Mr. *Gardiner* has shewn, in the explication of his Tables of Logarithms, a very easy method of getting the logarithm of any number whose last expressed figure, or figures, would recur infinitely perfect in the lowest place of the tables, upon the foundation of that figure or figures being numerator with 9,99,999, &c. denominators, that is, with as many nines as there are recurring figures.

The Double or Compound RULE *of* THREE.

IF the carriage of ½ a *C. wt.* 40 miles comes to 6*d.* what will be the charge of carrying 16 *C.* ¼ 100 miles?

First, *If* ,5—,025—16,25
,025

8125
3250

,5)40625(

Miles. *Miles.*
Then, *If* 40 ——— ,8125 ——— 100
100

40)81,2500(

Answer, 2,03125*l.*=2*l.* 0*s.* 7*d.*½.

Or thus,

C. l. C.
If ,5 ——— ,025 ——— 16,25
40 *Miles.* 100 *Miles.*

20,0 1625,00
,025

8125
3250

20)40,625

Answer, 2,03126*l.*=2*l.* 0*s* 7*d* ½

Or thus,

C. *Miles.* *l.*
If ,5 carried 40 comes to ,025

$$Q=\frac{,025\times100\times16,25}{,5\times40}=\frac{,05\times10\times16,25}{4}=2,03125$$

CHAP.

CHAP. IV.

Of PRACTICE.

THE fractions of some questions in this rule may with advantage be worked decimally; as,

1*st*. When the given price is just two shillings, 'tis only separating the right-hand figure for a decimal fraction, the rest are pounds;

EXAMPLE.

5295 lb. at 2*s*. *per* lb. *comes to* { 529,5 *or* 529*l*. 10*s*.

2*d*. When the price is any *aliquot* part of 2*s*. it is only separating the right-hand figure, as before, and then dividing the number by such part.

EXAMPLE.

oz. d.
258,3 at 8 *per oz.*

8*d* ⅓ 86,1 = 86*l*. 2*s*. *Answer.*

3*d*. When the given price can be divided into *aliquot* parts of two shillings, separate as before, and take such parts.

EXAMPLE.

	l. *d.*
	563,8 at 9 *per* lb.
d.	
$6\frac{1}{4}$	140,95
$3\frac{1}{2}$	70,475

Answer, 211,425=211*l.* 8*s.* 6*d.*

4*th*. When the price is any number of shillings whatsoever, 'tis only multiplying by their half (which is the decimal fraction for them) and the product is the answer.

EXAMPLES.

lb. *s.*	*oz.* *s.*
7296 at 12 *per* lb.	8264 at 15 *per oz.*
,6	,75
Ans. 4377,6=4377*l.* 12*s.*	41320
	57848
	Answer, *l.*6198,00

5*th*. The required price of any quantity may be found, by multiplying the said quantity by the decimal fraction answering to the price given.

EXAMPLE.

lb.   *l.*   *s.*   *d.*

8397 at 5 . 15 . 6 *per* lb.

5,675

41985
58779
50382
41985

*Ans.*47652,975=47652*l.* 19*s.* 6*d.*

6*th*. If

6*th*. If there are fractional parts in the given quantity, they must be reduced to decimals, as well as the fractional parts of the price.

EXAMPLES.

lb. s.	C. qrs. lb. l. s. d.
368½ at 13 *per* lb.	7368 . 3 . 14 at 1 . 12 . 6
368,5	
,65	7368,875
	1,625
18425	
22110	36844375
	14737750
Answer, 239,525	44213250
Equal to 239*l*. 10*s*. 6*d*.	7368875
	Answer, 11974,421875
	Equal to 11974*l*. 8*s*. 5*d*.¼.

The questions proposed in *Vulgar Arithmetic*, may serve for more examples.

CHAP. V.

Of INTEREST *and* REBATE.

THESE rules having been already sufficiently explain'd in *Vulgar Arithmetic*, it only now remains to shew how much more advantageously they may be work'd by decimal fractions. And, *first*,

SIMPLE INTEREST.

1*st.* THE annual intereſt of any ſum of money is found by only multiplying the given principal by the intereſt of one pound for a year; which,

at $\left\{\begin{matrix}4\\5\\6\end{matrix}\right\}$ *per Cent.* is $\left\{\begin{matrix},04\\,05\\,06\end{matrix}\right.$

found by dividing the rate of intereſt by 100.

EXAMPLES.

1*ſt.* What's a year's intereſt of 536 *l.* at 5 *l.* *per Cent*?
,05

Anſwer, 26,80=26*l.* 16*s.*

2*d.* What will the intereſt of 2367*l.* come to in a year, at 6*l.* *per Cent. per Annum*? ,06

Anſwer, 142,02*l.*=142*l.* 0*s.* 5*d.*

2*d.* Thus a year's intereſt of any ſum being found, 'tis only multiplying that ſum by 2, 3, 4, &c. and the product is the intereſt of the ſaid ſum, for 2, 3, 4, or more years.

EXAMPLE.

What's the intereſt of 623 *l.* for 3 years, at 4 *per Cent*?
,04

24,92 *Intereſt for* 1 *year.*
3

Anſwer, 74,76=74*l.* 15*s.* 2*d.* ½ *for* 3 *years.*

3*d.* Again, If the time required be months, 'tis only dividing the year's amount by ſuch parts as the given months are of a year.

EXAM-

EXAMPLES.

Ex. 1*st*. What is the interest of 539*l.* for four months, at 5*l.* *per Cent. per Annum?*

	l.	
	539	
	,05	
	26,95	*Interest for a year.*
Months.		
4 | $\frac{1}{3}$	8,9833=8*l.* 19*s.* 8*d.*	*Answer.*

Ex. 2*d.* What will the interest of 729*l.* 10*s.* 6*d.* come to in a year and 5 months, at 6 *per Cent. per Annum?*

Note, If there are any odd shillings, pence, and farthings, in the principal sum, they must be reduced to decimals; as in this example.

	729,525	
	,06	
Months	43,77150	*Interest for a year.*
4 | $\frac{1}{3}$	14,59050	*Interest for 4 months.*
1 | $\frac{1}{4}$	3,647625	*Interest for 1 month.*
Answer,	62,00962;	*Total Interest.*
	or	
	62*l.* 0*s.* 2*d.* $\frac{1}{4}$	

4*th*. But if it be required to find the interest of a sum of money for any number of days, you must either multiply the years amount by the number of days proposed, and divide the product by 365, or (which will be sometimes shorter) divide the year's amount by 365,

and

and multiply the quotient by the number of days proposed.

EXAMPLE.

What interest will 563*l.* amount to in 126 days, at 6*l.* *per Cent. per Annum?*

```
     l.
    563
    ,06
   -----
   33,78          365)33,78000(,09254
    126               --------
  ------               ..930
  20268                ------
  6756                  2000
 3378                   ----
 -------                 1750
365)4256,28(11,661       ----
   ----                   290
   .606                  -----
   ----                  ,09254
   2412                    126
   ----                  -----
    2228                 55524
   -----                18508
    .380                9254
   -----               --------
     .15               11,66004
```

But for the more ready casting up the interest of money for days, it will be necessary to have the interest of 1 *l.* for 1 day, at all rates ready calculated, as a table or standard. Thus:

The Interest *of* 1 l. *for* 1 *Day.*

At	1	*per Cent.* is	,00002739726
	2		,00005479452
	3		,00008219178
	4		,00010958904
	5		,00013698630
	6		,00016438356
	7		,00019178082
	8		,00021917808
	9		,00024657534
	10		,00027397260

The intereſt of 1*l.* for one day, is thus found; the given rate or intereſt of 100*l.* for a year, being divided by 100, quotes the intereſt of 1 *l* for a year; which again divided by 365, gives the required intereſt of 1*l.* for 1 day; which, when multiplied into both the number of days, and the principal ſum, produces the intereſt for the time required.

EXAMPLE.

What's the intereſt of 560*l.* for 60 days, at 5*l. per Cent. per Annum*?

```
,0001369863
        560 Principal.
-----------
   82191780
  6849315
-----------
,0767123280
         60 Days.
-----------
```

Anſwer, 4,6027396800=4*l.* 12*s.* 0*d.* ½.

The

The year being divided into 12 unequal months, the following verses may serve as a *memorandum* of the number of days in each, *viz.*

> *Thirty days hath* September,
> April, June, *and* November;
> February *hath twenty-eight alone,*
> *And all the rest have thirty-one.*

But without any farther trouble, the opposite table shews, by inspection only, the number of days from any day in any month, to the same day in any other month. As suppose the number of days, betwixt the 4th of *February* and the 4th of *September*, were required; looking in the column under *February* for *September*, against that month will stand 212, the number of days betwixt the said times: So again, from the 8th of *June* to the 8th of *October*, appears by the table to be 122 days, that being the number against *October*, in the column under *June*.

Note, If the given days are different, ('tis only adding, or subtracting their inequality to, or from the tabular number. Thus had the first example been from the 4th of *February* to the 8th of *September*, it had been 4 days more than 212, *viz.* 216; or had the time, in the last example, been from the 8th of *June* to the 4th of *October*, it had been 4 days less than 122, *viz.* 118.

Note also, If the time exceeds a year, 365 days must be added. Thus from the 4th of *February* 1758, to the 4th of *September* 1759, will be found to be 577 days, the sum of 212, and 365. See the Table.

Com-

To front Page 194.

A Table

Shewing the Number of Days from any day in any Month to the same day in any other Month.

From	Jan:	Feb:	Mar:	April.	May:	June:
To	Feb:31	Mar 28	April 31	May.30	June 31	July.30
	Mar:59	April 59	May.61	June.61	July.61	Aug:61
	April 90	May.89	June 92	July.91	Aug:92	Sep:92
	May 120	June 120	July 122	Aug:122	Sep:123	Oct:122
	June 151	July.150	Aug:153	Sep:153	Oct:153	Nov:153
	July.181	Aug:181	Sep:184	Oct:183	Nov:184	Dec:183
	Aug:212	Sep:212	Oct:214	Nov:214	Dec:214	Jan:214
	Sep:243	Oct:242	Nov:245	Dec:244	Jan:245	Feb:245
	Oct:273	Nov:273	Dec:275	Jan:275	Feb:276	Mar:273
	Nov:304	Dec:303	Jan:306	Feb:306	Mar:304	April 304
	Dec:334	Jan:334	Feb:337	Mar:334	April 335	May.334
	Jan:365	Feb:365	Mar:365	April 365	May.365	June.365

From	July.	Aug:	Sep:	Oct:	Nov:	Dec:
To	Aug:31	Sep:31	Oct:30	Nov:31	Dec:30	Jan:31
	Sep:62	Oct:61	Nov:61	Dec:61	Jan:61	Feb:62
	Oct:92	Nov:92	Dec:91	Jan:92	Feb:92	Mar:90
	Nov:123	Dec:122	Jan:122	Feb:123	Mar:120	April 121
	Dec:153	Jan:153	Feb:153	Mar:151	April 151	May 151
	Jan:184	Feb:184	Mar:181	April 182	May.181	June.182
	Feb:215	Mar:212	April 212	May 212	June 212	July.212
	Mar:243	April 248	May.242	June 243	July 242	Aug:243
	April.274	May.273	June 273	July 273	Aug:273	Sep:274
	May 304	June 304	July 303	Aug:304	Sep:304	Oct:304
	June.335	July 334	Aug:334	Sep:335	Oct:334	Nov:335
	July 365	Aug:365	Sep:365	Oct:365	Nov:365	Dec:365

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COMPOUND INTEREST.

1st. TO find the amount of any sum, at any rate of compound Interest, for any number of years.

RULE.

Multiply the amount of 1*l.* for a year, (which at 6 *per Cent.* is 1,06, at 5 *per Cent.* 1,05, &c.) so often into itself, as are the number of years proposed, wanting one; and the last product multiply'd by the principal, will give the amount required.

EXAMPLE.

What's the amount of 500*l.* forborn four years, at 4*l.* *per Cent per Annum?*

```
      1,04
      1,04
    ------
    1,0816
      1,04
   -------
  1,124864
      1,04
 ---------
1,16985856
       500
 ---------
```

Answer, 584,929280000*l.*=584*l.* 18*s.* 7*d.*

And thus is the first interest-table (which shews the amount of 1*l.* for any number of years, under 33, at the rates of 5 and 6 *per Cent. per Ann.*) form'd.

The use of which is plain and easy; for multiplying the figures standing against the number of years required, and under the given rate, by the proposed principal, the product is the amount desired.

Thus,

Thus, if the amount of 40*l.* in 20 years, at 5*l. per Cent. per Annum*, were required, 2,65329, the tabular number multiplied by 40*l.* the principal sum, gives 106,13160, or 106*l.* 2*s.* 7*d.*$\frac{1}{2}$, the *Answer*. And so of any other.

2*d.* To find the amount of any annuity, or yearly pension, forborn any number of years whatsoever, at any rate of compound interest.

RULE.

Multiply the first yearly payment by the amount of 1*l.* for 1 year, and to the product add the second yearly payment, the sum is the amount in 2 years; which, multiply'd again by the said amount, the product, with the addition of the third yearly payment, is the amount for 3 years, &c.

EXAMPLE.

What will a pension of 30*l.* *per Annum* amount to, being foreborn 4 years, at 5*l. per Cent. per Annum?*

	l.	
First yearly payment	30	
multiply'd by	1,05	
	31,50	
Second yearly payment added	30	
gives	61,50	*the amount in 2 years.*
	1,05	
	64,5750	
Third yearly payment	30	
	94,5750	*amount in 3 years.*
	1,05	
	99,303750	
Fourth yearly payment	30	
	129,303750	*amount in 4 years.*

 And

And after this manner is the second interest table (which shews the amount of 1 *l.* annuity for any number of years under 33, at the rates of 5 and 6 *l. per Cent. per Ann.* compound interest) compos'd.

But this second table may more easily be formed from the first, thus: the first line of this second table, or first year's amount being known, add of it the first line of the first table, the sum is the amount to 2 years, or the second line of this table; to which again adding the second line of the first table, you have the third of this, or the amount for three years, *&c.*

EXAMPLE, at 6*l. per Cent.*

l.		
1,	———	*First year of the second table.*
1,06	———	*First of the first table.*
2,06	———	*Second year of the second table.*
1,1236	———	*Second year of the first.*
3,1836	———	*Third year of the second.*
1,19101	———	*Third year of the first.*
4,37461	———	*Fourth year of the second,* &c.

The use of this second table is in the same manner as the first; as suppose the foregoing question for an example.

EXAMPLE.

	4,310112	*the annuity of* 1l. *annuity for* 4 *years, at* 5l. per Cent.
multiplied by	30	*the yearly sum.*
gives	129,30360	*as before.*

REBATE,

REBATE or DISCOMPT.

REBATE, according to simple interest, having been sufficiently explained in *Vulgar Arithmetic*, I shall only here set down one example work'd decimally the common way. And, to complete the section, add Mr. *Hutton*'s new method of finding the present worth and discompt of money, as delivered in his *System of Arithmetic*, pag. 188.

EXAMPLE.

What discompt must be allowed on a bill of 500*l.* paid 20 days before it is due, rebate at 5*l. per Cent. per Annum*? And what present money must be paid?

The common way.

	Days.	*l.*	Days.
If	365	5	20

5

365)100,0(,27397

2700

1450

3550

2650

95

Then

Then for the discompt.

If 100,27397 — ,27397 — 500
500

100,27397)136,98500(1,3661 *discompt.*

36711030

66288390

61240080

10756980

.629593

Or for the present worth.

If 100,27397 — 100 — 500
500

100,27397)50000,000(498,6338 *present worth.*

98904120

86575470

63562940

33985580

39033890

89516990

9297814

Mr. HATTON's *New Method.*

For the present worth, multiply the days in a year, the principal given, and 100 into each other, for a dividend; and add the product of 365 by 100 to that of the days multiplied into the rate given for the divisor; so the quotient arising is the answer. Thus

Thus in the foregoing example, 365×500×100 =18250000 for the dividend, and 365×100+20 ×5=36600 for the divifor.

See the work.

366|00)182500|00(498,6338 *prefent worth.*

```
 .3610
  3160
   2320
     240
     1420
      322
```

2. *For the difcompt,* multiply the rate principal, and days given together for a dividend; and proceeding as before directed for a divifor, the quotient will be the anfwer.

Thus keeping ftill the fame example, 5×500×20 =500060 for the dividend, and 365×100+20×5 =36600 for a divifor, as before.

See the work.

366|00)500|00(1,3661 *difcompt.*

```
 1340
  2240
   2240
    .440
     74
```

The

The truth of both these methods appears, not only by their agreement, but also by the addition of the present worth, and discompt, found; which, as near as can be, makes the principal.

Thus 498,6338 *the present worth,*
with 1,3661 *the discompt.*

makes 499,9999=500*l. the given principal.*

REBATE, *at* COMPOUND INTEREST.

1*st*. TO find the present worth of any sum to be paid at any number of years to come, rebate being made at any rate of compound interest.

RULE.

Divide the principal continually so many times by the amount of 1*l.* for 1 year, as are the number of years proposed, and the last quotient is the answer.

EXAMPLE.

What is the present worth of 1*l.* to be paid at the end of 6 years, rebate at 6*l. per Cent. per Ann.* compound interest?

	l.				*Years.*
The principal	1,		,943396		1
	,943396	*divi-*	,889997	*present*	2
	,889997	*ded by*	,839619	*worth*	3
	,839619	1,06	,792094	*at the*	4
	,792094	*gives*	,747258	*end of*	5
	,747258		,704960		6

And thus is the third table made, which shews the present worth of 1*l.* due at any number of years to come under 33, rebate at 5 and 6*l. per Cent. per Annum,* compound interest; the use of which is as the

the other tables, only by multiplying the present worth of 1*l.* by the given principal, and the product is the present worth required.

Thus, suppose in the example above, the principal had been 200*l.* the present worth of 1*l.* *viz.* ,704960 multiply'd by 200, gives 140,992000, the present worth of 200*l.* and so any other.

2*d.* To find the present worth of any annuity, or yearly pension, to continue any number of years; rebate being made at any rate, *per Cent. per Ann.* compound interest.

RULE.

Find by the foregoing rule the present worth of the propos'd annuity, for 1, 2, 3, or so many years as are demanded; and the sum of those respective present worths will be the value of the annuity; that is, the first and second of those values, added together, will be the present worth for two years; and the first, second, and third, added for three years, &c.

EXAMPLE.

What's the present worth of an annuity of 50*l.* to continue four years, rebate at 5*l.* *per Cent. per Annum?*

l.			
50,	*divided by*	1,05	47,61904
47,61904	*gives,*		45,35147
45,35147			43,19187
43,19187			41,13512

The total of which is 177,29750 *the present worth requir'd.*

And after this manner is made table the fourth, which shews the present worth of 1*l.* annuity, to continue any number of years under 33, rebate at 5 and

Decimal Tables,

Shewing

TABLE 1st. The Amount of 1£. for any number of Years under 33 at the Rates of 3 & 4 PC.t P An. Com. Int

TABLE 2d. The Amount of 1£. Annuity for any number of Years under 33 at the Rates of 3 & 4 PC.t P An. C. Int.

Rates 3	Rates 4	Years	Rates 3	Rates 4
1,03000	1,04000	1	1,00000	1,00000
1,06090	1,08160	2	2,03000	2,04000
1,09273	1,12486	3	3,09090	3,12160
1,12551	1,16986	4	4,18363	4,24646
1,15927	1,21665	5	5,30914	5,41632
1,19405	1,26532	6	6,46841	6,63297
1,22987	1,31593	7	7,66246	7,89829
1,26677	1,36857	8	8,89233	9,21423
1,30477	1,42331	9	10,15910	10,58279
1,34392	1,48024	10	11,46388	12,00610
1,38423	1,53945	11	12,80780	13,48635
1,42576	1,60103	12	14,19203	15,02580
1,46853	1,66507	13	15,61780	16,62684
1,51259	1,73167	14	17,08632	18,29191
1,55797	1,80094	15	18,59891	20,02359
1,60471	1,87298	16	20,15688	21,82453
1,65285	1,94790	17	21,76159	23,69751
1,70243	2,02582	18	23,41443	25,64541
1,75350	2,10685	19	25,11687	27,67123
1,80611	2,19112	20	26,87037	29,77808
1,86029	2,27877	21	28,67649	31,96920
1,91610	2,36992	22	30,53678	34,24797
1,97359	2,46472	23	32,45288	36,61789
2,03279	2,56330	24	34,42647	39,08260
2,09378	2,66583	25	36,45926	41,64591
2,15659	2,77247	26	38,55304	44,31174
2,22129	2,88337	27	40,70963	47,08421
2,28793	2,99870	28	42,93092	49,96758
2,35656	3,11865	29	45,21885	52,96629
2,42726	3,24340	30	47,57542	56,08494
2,50008	3,37313	31	50,00268	59,32833
2,57508	3,50806	32	52,50276	62,70147

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Decimal Tables,

Shewing

TABLE 3d. The present Worth of 1£ due at any Number of Years under 33 Rebate at 3 & 4 P Ct. P An. Com. Int.			TABLE 4th. The present Worth of 1£ Annuity for any Number of Years under 33. Rebate at 3 & 4 P Ct. P Ann.	
Rates 3	Rates 4	Years	Rates 3	Rates 4
,970874	,961538	1	0,97087	0,96154
,942596	,924556	2	1,91347	1,88609
,915142	,888996	3	2,82861	2,77509
,888487	,854804	4	3,71710	3,62989
,862609	,821927	5	4,57971	4,45182
,837484	,790314	6	5,41719	5,24214
,813091	,759918	7	6,23028	6,00205
,789409	,730690	8	7,01969	6,73274
,766416	,702587	9	7,78611	7,43533
,744094	,675564	10	8,53020	8,11090
,722421	,649581	11	9,25262	8,76048
,701380	,624597	12	9,95400	9,38507
,680951	,600574	13	10,63496	9,98565
,661118	,577475	14	11,29607	10,56312
,641862	,555264	15	11,93793	11,11839
,623167	,533908	16	12,56110	11,65230
,605016	,513373	17	13,16612	12,16567
,587394	,493628	18	13,75351	12,65930
,570286	,474642	19	14,32380	13,13394
,553676	,456387	20	14,87747	13,59033
,537550	,438833	21	15,41502	14,02916
,521892	,421955	22	15,93692	14,45112
,506692	,405726	23	16,44361	14,85684
,491934	,390121	24	16,93554	15,24696
,477606	,375117	25	17,41315	15,62208
,463695	,360689	26	17,87684	15,98277
,450189	,346817	27	18,32703	16,32959
,437077	,333477	28	18,76411	16,66306
,424346	,320651	29	19,18845	16,98371
,411987	,308319	30	19,60044	17,29203
,399987	,296460	31	20,00043	17,58849
,388337	,285058	32	20,38877	17,87355

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Decimal Tables

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TABLE 1st. The Amount of one Pound for any Number of Years under 33, at ye Rates of 5 & 6 p Ct. p An: Com: Inter:

TABLE 2d. The Amount of 1£ Annuity for any number of Years under 33, at ye Rates of 5 & 6 pct. p an: Com: Interest.

Rates 5	Rates 6	Years	Rate 5	Rate 6
1,05000	1,06000	1	1,00000	1.00000
1,10250	1,12360	2	2,05000	2.06000
1,15762	1.19101	3	3,15250	3.18360
1,21550	1.26247	4	4,31012	4.37461
1,27628	1.33822	5	5,52563	5.63709
1,34009	1.41852	6	6,80191	6.97532
1.40710	1.50363	7	8,14200	8.39383
1,47745	1.59384	8	9.54910	9.89746
1.55132	1,68948	9	11.02656	11.49131
1.62889	1.79084	10	12,57789	13.18079
1,71034	1,89829	11	14.20678	14.97164
1,79585	2.01219	12	15.91712	16.86994
1.88565	2.13292	13	17.71298	18.88213
1.97993	2.26090	14	19.59863	21.01506
2.07892	2.39655	15	21.57856	23.27597
2,18287	2.54035	16	23.65749	25.67252
2.29201	2.69277	17	25.84036	28.21288
2.40662	2.85434	18	28.13238	30.90565
2,52695	3.02559	19	30.53900	33.75999
2,65329	3,20713	20	33.06595	36.78559
2,78596	3.39956	21	35.71925	39.99272
2.92526	3.60353	22	38.50521	43.39229
3,07152	3.81975	23	41.43047	46.99582
3.22510	4.04893	24	44.50199	50.81557
3,38635	4.29187	25	47.72709	54.86451
3.55567	4.54938	26	51.11345	59.15638
3.73345	4.82234	27	54.66912	63.70576
3,92013	5.11168	28	58.40258	68.52811
4.11613	5.41838	29	62.32271	73.63979
4.32194	5.74349	30	66.43884	79.05818
4.53804	6.08810	31	70.76079	84.80167
4.76494	6.45338	32	75.29883	90.88977

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Between Pages 202 & 203.

Decimal Tables

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TABLE 3d. The present Worth of 1 £ due at any number of years to come under 33. Rebate at 5 & 6 ℔ C.t ℔ an: Com: Interest

TABLE 4th. The present Worth of 1 £ Annuity to continue any number of Years under 33. Rebate at 5 & 6 ℔ C.t ℔ an: Com: Interest

Rates 5	6	years	Rates 5	6
.952381	.943396	1	0.95238	0.94339
.907030	.889996	2	1.85941	1.83339
.863838	.839619	3	2.72324	2.67301
.822702	.792093	4	3.54595	3.46510
.783526	.747258	5	4.32947	4.21236
.746215	.704960	6	5.07569	4.91732
.710682	.665057	7	5.78637	5.58238
.676839	.627412	8	6.46321	6.20979
.644609	.591898	9	7.10782	6.80169
.613913	.558394	10	7.72173	7.36008
.584679	.526787	11	8.30641	7.88687
.556837	.496969	12	8.86325	8.38384
.530321	.468839	13	9.39357	8.85268
.505068	.442301	14	9.89864	9.29498
.481017	.417265	15	10.37965	9.71225
.458111	.393647	16	10.83777	10.10589
.436296	.371364	17	11.27406	10.47726
.415520	.350343	18	11.68958	10.82760
.395734	.330513	19	12.08532	11.15811
.376889	.311804	20	12.46221	11.46992
.358942	.294155	21	12.82115	11.76407
.341849	.277505	22	13.16300	12.04158
.325571	.261797	23	13.48857	12.30338
.310067	.246978	24	13.79864	12.55035
.295302	.232998	25	14.09394	12.78335
.281240	.219810	26	14.37518	13.00316
.267848	.207368	27	14.64303	13.21053
.255093	.195630	28	14.89812	13.40616
.242946	.184556	29	15.14107	13.59072
.231377	.174110	30	15.37245	13.76483
.220359	.164254	31	15.59281	13.92908
.209865	.154956	32	15.80267	14.08404

Webster sculp. Bickham [illegible]

and 6*l. per Cent, per Ann.* compound intereſt. Which table, as the others, will give the preſent worth of any ſum whatſoever, by only multiplying the value of 1*l.* by the ſum propoſed. Thus, ſuppoſe the foregoing example.

E X A M P L E.

3,54595 *the preſent worth of* 1l. *per an. for* 4 *years.*
multiply'd by 50 *the annuity propoſed.*

gives 177,29750 *as before.*

The fourth table may be alſo form'd from the third, thus:

The firſt year is the ſame in both; the firſt and ſecond years of the third, added together, make the ſecond of the fourth; the ſecond year of the fourth, added to the third year of the third, makes the third of the fourth table, &c. See the example.

E X A M P L E, at 6l. *per Cent.*

,94339 * *The firſt year of both tables.*
,88999 *The ſecond year of the third table.*

1,83339 *The ſecond year of the fourth.*
,83961 *The third year of the third.*

2,673000 *The third year of the fourth.*
&c.

* *Note.* 1 is added to the right-hand figure, in conſideration of the places omitted.

More

More EXAMPLES *for the exercise of the foregoing* Rules and Tables.

Ex. 1*st*. What will 1000*l* increase to, forborn 22 years, at 5 *per Cent. per Annum* compound interest?

Answer, 2925*l*. 5*s*. 2*d*.$\frac{1}{2}$.

Ex. 2*d*. What would an annuity of 520*l*. forborn 17 years, at 6 *per Cent*. compound interest, amount to?

Answer, 14670*l*. 13*s*. 11*d*.$\frac{1}{2}$.

Ex. 3*d*. What present money would discharge a debt of 7500*l*. to be paid at the end of ten years, rebate being made at 5*l*. *per Cent. per Annum*?

Answer, 4604*l*. 6*s*. 11*d*.$\frac{1}{4}$.

Ex. 4*th*. What is the yearly rent of 65*l*. to continue 30 years, worth in ready money, rebate being made at 6 *per Cent. per Annum* compound interest?

Answer, 894*l*. 14*s*. 3*d*.$\frac{1}{4}$.

Ex. 5*th*. What is the present worth of a reversion of a lease of 500*l*. *per Annum*, to continue 20 years, but not to commence till after the end of five years, allowing the purchaser 6 *per Cent. per Annum* compound interest?

Note, To work questions of this nature, there must be two distinct operations, *viz*

1*st*. The present worth of the proposed annuity; for the given time of its continuance must be found as if it were immediately to commence. Then,

2*d*. See what principal, forborn at the given interest, would, in the time to the commencing of the reversion, amount to the aforesaid present worth, and that principal will be the present worth of such annuity in reversion.

Thus

Thus by table the 4th, the present worth of 1*l.* per Annum, *to continue* 20 *years, at* 6*l.* per Cent. *is found to be.*

	11,46992
which multiplied by the proposed annuity,	500
gives	5734,9600'0

Then, in the first table, against 5 years, the time to the commencing of the reversion, under the same rate of interest, is found 1,33822, the amount of 1 *l.* in the said time; therefore,

If 1,33822 *arise from* 1*l. from what principal does* 5734,9600 *arise?*

Answer, 4285,5135=4285*l.* 10*s.* 3*d.* ¼.

Or it may more easily be done by first finding the present worth of the annuity for the whole time, both of possession and reversion, and then deducting the present worth of the possession; the remainder will be the present worth of the reversion. Thus 12,78335, the tabular number against 25 years, the whole time being multiplied by 500, gives 6391,67500; from which subtracting 2106,18000, the produce of 4,21236 the tabular number against 5 years, the time of the possession, the remainder 4285,49500, is the value of the reversion; Within a trifle the same answer as before.

FREEHOLD ESTATES.

TO find the present worth of any annual rent, to continue for ever, commonly called fee-simple.

 RULE.

RULE.

Divide the proposed rent by the interest of 1*l.* for one year, (which at 5*l. per Cent.* is ,05, or at 6 *per Cent.* ,06, as before shewn) and the quotient is the present value of the estate.

EXAMPLE.

What is an estate of 200*l. per Annum*, to continue for ever, worth in ready money, allowing the purchaser 5*l. per Cent. per Annum*, compound interest?

—,05)200,00(
Answer, 4000*l.*

Or it may be done by multiplying the fee-simple of 1*l.* by the yearly rent proposed.

The fee-simple of 1*l. per Annum*, compound interest, at

1 *per Cent.*	2 *per Cent.*	3 *per Cent.*	4 *per Cent.*	5 *per Cent.*
100,00000	50,0000	33,33333	25,00000	20,00000

6 *per Cent.*	7 *per Cent.*	8 *per Cent.*	9 *per Cent.*	10 *per Cent.*
16,66667*	14,28571	12,50000	11,11111	10,00000.

**Note*, In contracting a line of decimals from many to fewer places, 1 is added to the last figure retained, if the next omitted figure exceedeth 5; which rule hath been all along observed in the tables, &c.

The foregoing question worked by the last rule.

	l.
The fee-simple *of* 1 l. *per Ann. at* 5 l. *per Cent. is*	20
which, multiplied by the yearly rent,	200
gives, as before,	4000

I will

I will now only add another example or two, for the reader's exercise, and conclude this rule.

1. *A.* has the possession of an estate of 130*l.* *per Annum*, to continue 20 years; *B.* has the reversion of the same, from that time for ever. What must *A.* give *B.* if he would purchase his reversion? And what must *B.* give *A.* if he would buy his possession, accounting 6 *per Cent.* compound interest in each case?

Answer, { A's *possession is worth* . 1491 . 1 . $9\frac{1}{4}$
B's *reversion is worth* . 675 . 11 . $6\frac{1}{2}$ }

2. *A.* agrees with *B.* for an annuity of 600*l.* *per Annum*, to continue 25 years, to give him the present worth of it, at 6*l.* *per Cent.* *per Annum*; but not having money enough by him, offers to make over to him a freehold estate of 12 *l.* *per Annum*, at the same interest: What money besides will pay his purchase?

Answer, 7470 *l.* 0*s.* 2*d.* $\frac{1}{2}$.

CHAP. VI.

Of FELLOWSHIP.

WITHOUT taking any notice of the common way of working this rule, observe this much more compendious method.

RULE.

Divide the whole gain, or loss, by the whole stock, and multiply the quotient by each man's particular stock; the several products are the respective gains of each.

Note, It is necessary to place so many cyphers on the right hand of your dividend, as will bring out six or seven fractional places in the quotient: but if the last, or any figure of it will recur infinitely, put down the vulgar fraction in that place; and, to make up the several products, do as shewn in the indirect Rule of Three.

EXAMPLE.

Suppose *A*. *B*. and *C*. trading together, *A* puts in 500*l*. *B*. 700*l*. and *C*. 1200*l*. the whole gain is 836*l*. What share of it belongs to each?

$$\left.\begin{array}{l} A = 500 \\ B = 700 \\ C = 1200 \end{array}\right\} = 2400)836,000\,0(.348\tfrac{8}{24} = .348\tfrac{1}{3}$$

$$\begin{array}{r} .348\tfrac{1}{3} \\ 500 \\ \hline \end{array} \qquad \begin{array}{r} .348\tfrac{1}{3} \\ 700 \\ \hline \end{array} \qquad \begin{array}{r} .348\tfrac{1}{3} \\ 1200 \\ \hline \end{array}$$

$$\left.\begin{array}{l} A\text{'s share} = 174\ 1\tfrac{2}{3} \\ B\text{'s} \ldots = 243.8\tfrac{1}{3} \\ C\text{'s} \ldots = 418.0 \end{array}\right\} \begin{array}{l} B\text{'s} = 243.8\tfrac{1}{3} \quad C\text{'s} = 418.0 \\ = 836l.\ \text{the total gain.} \end{array}$$

See

See another EXAMPLE *with time.*

A, puts into company 525*l.* 10*s.* for six months. *B*. 382*l.* 15*s.* for eight months; and *C*. 1000*l.* for four months; they gain in all 1286*l.* 12*s.* How much is that for each?

	525,5	382,75	1000
	6	8	4
A's *stock*	3153,0	3062,00	4000
B's ——	3062		
C's ——	4000		

10215)1286,600000(,125952

	,125952	,125952	,125952
	3153	3062	4000
A's *gain*	397,126656	385,665024	503,808000
B's ——	385,665024		
C's ——	503,808000		

Total, 1286,599680, *or* 1286*l.* 12*s.* *nearly*.

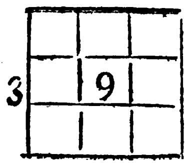
CHAP. VII.

Of EVOLUTION, *or* EXTRACTION *of* ROOTS.

The SQUARE ROOT.

EXTRACTION of the SQUARE ROOT is finding the ſide of a ſquare figure; or, numerically ſpeaking, it is finding what number, multiplied by itſelf, will produce the number given. Thus the ſquare root of 16 is 4, 4 times 4 making 16.

What a ſquare is, may be ſeen in the following figure: which being divided every way into 3 equal parts, its whole content or ſquare is 9, and its ſide or root is 3.



Square

Square numbers are either single or compound.

A single square number is always less than 100, being produced by the multiplication of some one single figure by itself; as 16 from 4, &c. So that the root of any single square may be found in the annexed table. always taking the root of the next less square, for any number not there inserted; as, for 26 take 5, or 3 for 10, &c.

Square	1	4	9	16	25	36	49	64	81
Roots	1	2	3	4	5	6	7	8	9

A compound square number being composed by the multiplication of two or more figures by themselves, always exceeds 100; as 144, which is 12 times 12; or 225, which is 15 times 15, &c.

If the root therefore is express'd by two figures, its square must at least consist of three; for the least root express'd by two figures is 10, whose square is 100. And if the root has three figures, its square must at least have five. If four, the square has seven, &c. So that you cannot augment the root one figure, but you increase the square two.

To find the Root of any compound square Number; as suppose 2704.

1*st*. You must distinguish it into single squares, by placing dots over every other figure, beginning at the right-hand thus;

$$\dot{2}7\dot{0}4$$

And so many dots as happen, so many places will the root consist of; which in this example are two.

2dly. Drawing a crooked line on the right-hand of your number, as in division, find the root of your first single square, and place it in the quotient.

See the work.

2704(5

3dly. Placing the square of the root found, under the first single square, subtract, and set down the remainder; bringing down to it the next single square, call the line a *Resolvend*.

2704(5
25
———
204 *Resolvend*.

4thly. Drawing another crooked line on the left-hand of the said resolvend, place beyond it the double of the quotient, in form of a divisor,

2704(5
25
———
10)204

5thly. Dividing the resolvend, all but the unit's place, by the said divisor (which, in this example, is the 10's in 20) set down the number of times it goes (*viz*. 2) both in the quotient, and on the right-hand of the divisor.

2704(52
25
———
102)204 *Resolvend*.

6thly.

6thly. Multiplying the whole divisor by the figure last placed in the quotient, set down the product under and subtract it from the resolvend.

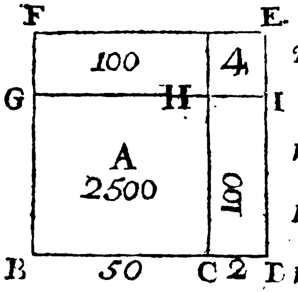
See the work:

```
 . .
2704(52
25
------
102)204
    204
   ----
     .0
```

Note, If the divisor multiply'd by the quotient-figure, gives a product greater than the resolvend, 'tis false, and must be rectified by a smaller quotient-figure.

Extraction of the square root being a matter rather lineal than numeral, the reason of the rules above given for its operation, will best appear from the following figure: Wherein let it be considered, that if the root *B*. *C*. of the square *A*, be augmented by the magnitude *C*. *D*. the said square will be inlarged by the figure *C*. *D*. *E*. *F*. *G*. *H*. *I*. equal to an oblong or long square, whose length shall be the root *B* *C*. twice taken, with the line *C*. *D*. and breadth the same line *C*. *D*.

N. *B*. The particular contents of the several parts of the figure are as therein set down.



The several parts of the Square,	2500 100 100 4
Whole square,	2704
Parts of the Root,	50 2
Whole Root,	52

In the operation therefore, having taken the nearest root of the first square 27, *viz.* 5, and placed it in the quotient, from the place it stands in (*viz.* the second) it must be accounted 50; the square of which, *viz.* 2500 (marked *A.* in the figure) being subtracted from the given number 2704, there remains 204, equal to the figure *C. D. E. F. G. H. I.* In order to find the content of which, I double the root *B. C. viz.* 50, for the two bases *C. H.* and *H. G.* which making 100, I set it down as a divisor; and to find the height of the said figure, *viz. C. D.* I seek how often the said divisor, is contain'd in the number 204; and finding it twice, I set down two in the quotient for the said height and also in the unit's place of my divisor for the base *H. I.* Then multiplying the bases by the height, I have the content of the whole annexed figure *C. D. E. F. G. H. I.* *viz.* 204, exactly equal to the number remaining, after the subtraction of the first great square 2500; so that the number given, *viz.* 2704, appears to be a perfect square, whose root or side is exactly 52.

Note, As many single squares as are in the number proposed, so many times must the work of the three last rules be repeated; every square being to be brought down, as directed in the third rule.

EXAM-

EXAMPLE.

What's the root of 582169(763 *Anſwer.*
49
146).921 *Reſolvend.*
876

1523).4569 *Reſolvend.*
4569

0

In this example, to 45 the ſecond remainder, 69 the next ſquare, being brought down, makes 4569 for a new reſolvend ; and 76, the quotient-figures, being doubled for a new diviſor, makes 152, which going three times in the reſolvend, 3 is placed both in the quotient, and on the right-hand of the diviſor, which being then multiplied by 3, gives 4569, equal to the reſolvend, ſo that nothing remains.

PROOF.

The Proof of this extraction, is only multiplying the root found by itſelf; which if right will produce the ſquare number given. *Note,* if any thing remains, it muſt be taken in.

To find the fractional Part of the Root.

Add to the number given a competent number of pairs of cyphers, and proceed by the foregoing rules: As ſuppoſe 75896 were given for an example.

EXAMPLE.

```
       75896,000000(275,492
       4
       ---
     47)358
        329
        ---
    545).2996
         2725
         ----
   5504).27100
         22016
         -----
  55089).508400
          495801
          ------
 550982).1259900
          1101964
          -------
          .157936
```

Note, As many pairs of cyphers as are added, so many fraction places of decimal parts will be in the root.

To extract the SQUARE ROOT *of a* VULGAR FRACTION,

FIND the root of the numerator for a numerator; and let the root of the denominator be its denominator; thus the square root of $\frac{16}{25}$ is $\frac{4}{5}$, and of $\frac{4}{9}$ is $\frac{2}{3}$, *&c.*

The root of a mixed number is also found after the same manner, being first reduced into an improper fraction;

fraction; thus $6\frac{1}{4}$, reduced, makes $\frac{25}{4}$, the root of which is $\frac{5}{2}$, or $2\frac{1}{2}$.

But if either a proper fraction, or a mixed number, be incommensurable to its root; to extract it, reduce the fraction to a decimal of an even number of places, and extract the root as if it were a whole number: thus $\frac{7}{8}$, reduced to a decimal, will be ,8750, the square root of which is ,935, &c. but had it been $4\frac{7}{8}$, which reduced is 4,8750, the root would be 2,20794, &c.

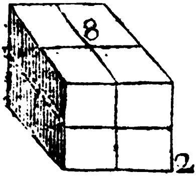
The CUBE ROOT.

EXtraction of the CUBE ROOT is finding the side of a solid figure, whose length, breadth, and depth are equal; or numerically speaking, it is finding what number multiply'd into itself, and then into the product, will produce the number given; thus the Cube Root of 64 is 4, 4 times 4 being 16, and 4 times 16 make 64.

What a Cube is may be something explain'd by the following figure; wherein 8 dice are so disposed, that there are two every way, that is two in length, 2 in breadth, and 2 in height; so that 8 is the cube, and 2 the root.

See

See the figure.



Cube numbers are either ſingle or compound.

A ſingle cube number is always leſs than 1000, being produced by the multiplication of one ſingle figure, firſt by itſelf, and then by its product; as 125 from 5, &c. So that the root of any ſingle cube may be found in the annexed table, always remembring to take the root of the next leſs cube for any number not there inſerted, as for 513 take 8, or 6 for 220, &c.

Cubes	1	8	27	64	125	216	343	512	729
Squrs.	1	4	9	16	25	36	49	64	81
Roots.	1	2	3	4	5	6	7	8	9

A compound cube-number, being compoſed by the multiplication of 2 or more figures, firſt by themſelves, and then by their product, always exceeds 1000, as 1728 from 12, or 15625 from 25, &c.

If therefore the root is expreſs'd by two figures, its cube muſt at leaſt conſiſt of four; for the leaſt root expreſſed by two figures is 10, whoſe cube is 1000; if the root has three figures, its cube muſt at leaſt have ſeven, &c. So that you cannot augment the root one figure, but you increaſe the cube three.

There-

Therefore, to find the root of any compound cube-number, as suppose 15625.

1*st*. You must distinguish it into single cubes, by placing dots over every third figure, beginning at the right-hand, thus :

$$\dot{1}5\dot{6}25$$

And so many dots as happen, so many places will the root consist of; which, in this example, are two.

2*dly*, Drawing a crooked line on the right-hand of your number, as in division, set down as a quotient the root of your first single cube; which in this example, is 2.

3*dly*, Placing 8 the cube of 2, the root found, under 15 the first single cube, subtract, and to the remainder 7 bring down 625, the next single cube, and it will make 7625; which call a Resolvend.

See the work.

$$\begin{array}{l} \dot{1}5\dot{6}25(2 \\ \underline{8} \\ 7625 \; \textit{Resolvend.} \end{array}$$

4*thly*, Draw a line under the resolvend, and tripling the square of the root 2, set the said triple square (*viz.* 12) under the resolvend; so that units in the said triple square may stand under the place of hundreds in the resolvend.

5*thly*,

5thly, Subſcribe alſo the triple of the root 2, (*viz.*6) ſo that units, in this, may ſtand under the place of tens in the reſolvend.

6thly. The triple-ſquare of the root, and triple root being placed as directed, draw a line under them, and add them together in the order they are placed; the ſum 126 is a diviſor.

7thly. Accounting all the reſolvend, except the place of units, (*viz*. 762) a dividend, ſeek how often the diviſor (126) is contained in it, and place the number of times (which is here 5) in the quotient.

8thly. Draw a line under the diviſor, and multiplying the triple ſquare (12) by 5, the figure laſt placed in the quotient, ſet the product (60) ſo under the ſaid triple ſquare, that units may ſtand under units, and tens under tens.

9thly. Squaring the figure (5) laſt placed in the quotient, multiply its ſquare (*viz*. 25) by the triple root (6), and place the product (150) ſo, that units, in this may ſtand under units in the ſaid triple number.

10thly. Subſcribe the cube of (5), the figure laſt placed in the quotient, which will be 125; ſo that tens, in this, may ſtand under units in the former product.

11thly. Then drawing a line, add the three numbers laſt placed together; and ſubtracting the ſum 7625 (which is called the *ablatitium*) from the reſolvend, ſet down the remainder (if any) in order underneath, as in common ſubtraction.

See the work.

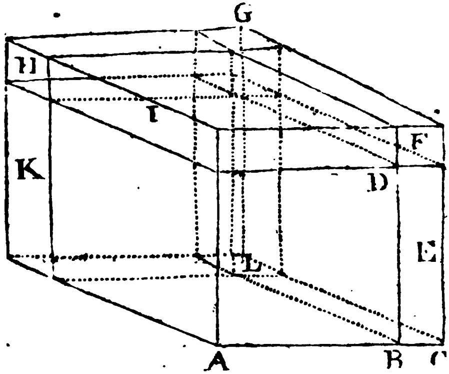
$$
\begin{array}{ll}
\dot{1}5\dot{6}25(25 & \\
8 & \text{Cube of } 2. \\
\hline
7625 & \text{Resolvend.} \\
\hline
12 & \text{triple square of } 2. \\
6 & \text{triple of the Root } 2. \\
\hline
126 & \text{Divisor.} \\
\hline
60 & \text{triple square} \times 5. \\
150 & \text{triple Root} \times 5 \times 5. \\
125 & \text{Cube of } 5. \\
\hline
7625 & \text{Ablatitium.} \\
\hline
\ldots 0 &
\end{array}
$$

Note, If the given number hath more places, the next single cube must be brought down to the last remainder, for a new resolvend; and the work of the 4th, 5th, 6th, 7th, 8th, 9th, 10th, and 11th rules, must be repeated as often as you so form a new resolvend.

Note also, if the *ablatitium* is greater than the resolvend, the work is false, and must be rectified by placing a lesser figure in the quotient.

To account for these rules of extracting the cube root, we must again make use a figure with literal references, as follows:

From



From whence it appears, That if the root *A. B.* of the cube *A. D.* be augmented by the magnitude *B. C.* the cube is thereby enlarged by the addition of seven parallelepipedes,* *viz.* 1*st.* The little cube *G*, whose root is the line *B. C.* 2*d.* The three parallelepipedes, mark'd *I. K. E.* which have the square of the root *A. B.* for base, and the line *B. C.* for height. 3*d.* The three parallelepipedes mark'd *F. H. L.* which have the square of the line *B. C.* for base, and the root *A. B.* for height.

In the operation therefore of the foregoing example, 15625 being divided into single cubes, it appears that the root will consist of two places: The nearest root then of the first cube 15, *viz.* 2, being placed in the quotient where its value will be 20, from the place it stands in, and its cube 8000 subtracted from the given number 15625, the remainder 7625, will be equal to the seven parallelepipedes, *G. I. K. E. F. H. L.* the bases of three of which, *viz. I. K. E.* being the square of the root *A. B.* three times the square of 20, *viz.*

---

* *Parallelepipedes is a geometrical term, signifying regular figures, whose opposite sides and angles are equal.*

viz. 1200 is set down under the said remainder, or resolvend; also three more of the said parallelepipedes, *viz. F. H. L.* having their height equal to the said root three times 20, *viz.* 60, the sum of the three heights, is added to the said 1200, the sum of the bases of the other three, for a divisor; by which dividing the resolvend, the number of times it goes, *viz.* 5, is the height of the three parallelepipedes *I. K. E.* and the root of the square of the bases of the other three parallelepipedes *F. H. L.* and also the root or side of the parallelepipedon, or little cube *G.* Multiply therefore 1200 by 5, for the whole contents of the parallelepipedes *I.K.E. viz.* 6000; and squaring the said 5 for the bases of the three parallelepipedes *F. H. L.* multiply their three heights 60, by the said square of their bases, the product is 1500, their whole contents; under which set down also the cube of 5, *viz.* 125, for the content of the parallelepipedon *G.* the sum of these three numbers I find to be just 7625, exactly equal to the resolvend; which shews the number given to be a perfect cube, whose root is exactly 25.

To make all more plain, see the work at length, with the vacant places fill'd with cyphers.

15625	20
8000 *Cube* A. D.	5
7625 *Remainder, or Resolvend.*	25 *Root.*

1200 *Bases of Parallelepipedes* I. K. E.
60 *Height of Parallelepipedes* F. H. L.

1260 *Divisor.*

6000 *Content of Parallelepipedes* I. K. E.
1500 *Content of Parallelepipedes* F. H. L.
125 *Content of Parallelepipedon* G.

7625 *Total, or Ablatitium.*

...0

PROOF.

PROOF.

The proof of this extraction is, by multiplying the root found, first by itself, and that product again by the root; adding the remainder, if there be any, to the last product: thus,

	25 *the Root.*
multiply'd by	25
	125
	50
gives	625
again by	25 *the Root.*
	3125
	1250
gives	15625

To find the fractional part of the Root.

Add to the number given, so many ternaries of cyphers, as you would have places of decimal parts in the root, and proceed by the foregoing rules. As suppose 865635 were given for an example.

EXAM-

EXAMPLE.

865	635	000	000	000	(95,304 *Root.*
729					
136	635	—	—	—	1st *Resolvend.*
24	3				
	27				
24	57	—	—	—	*Divisor.*
121	5				
6	75				
	125				
128	375	—	—	—	*Ablatitium.*
8	260	000	—	—	2d *Resolvend.*
2	707	5			
	2	35			
2	710	35	—	—	*Divisor.*
8	122	5			
	25	65			
		27			
8	148	177	—	—	*Ablatitium.*
	111	823	000	—	3d *Resolvend.*
	272	462	7		
		28	59		
	272	491	29	—	*Divisor.*
	111	823	000	000	4th *Resolvend.*
	27	246	270	0	
			285	90	
	27	246	555	90	*Divisor.*
	108	985	080	0	
		4	574	40	
				64	
	108	989	654	464	*Ablatitium.*
	2	833	345	536	*Remainder.*

In

In this example, there being three ternaries of cyphers, the root must have three fractional places.

Note, The divisor going no times in the third resolvend, the same figures are immediately brought down again, and with a new ternary of cyphers form the 4th resolvend.

To extract the Cube Root of a vulgar Fraction.

Find the root of the numerator given for a new numerator, and the root of the denominator given for a new denominator; thus the cube root of $\frac{64}{125}$ is $\frac{4}{5}$.

But if the vulgar fraction be incommensurable to its root, reduce it to a decimal of three, six, nine, or any ternary of places, and then extract its root.

CON-

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VULGAR ARITHMETIC.

 Chap.